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# Optimal bidding for a bundle of power transmission infrastructure works

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ABSTRACT

Motivated by recent reforms in Chile's electricity market, where bundled auctions are used to attract participation and accelerate investment, we develop a theoretical model that explains the strategic role of private financing conditions in such settings. We put forward an optimal bidding mechanism for a bundle of power transmission infrastructure works. Specifically, the regulator auctions two works altogether: one is to be developed and operated by the winning bidder, while the other is an owner-operated and financed expansion of an existing work. Participants bid jointly for both contracts, and the package is awarded based on the lowest total bid. The costs are divided into a common developing part for all participants and a private part related to financing. The optimal bidder offers the expected value of the costs, adjusted for the cost advantage over the second lowest bidder. This approach efficiently allocates the works to the firm with the lowest combined costs. However, rents persist due to the informational and cost advantage in financing. When a bidder expects higher costs, it requests a higher payment, which reduces its chances of winning the bid.

## 1. Introduction

Electric power transmission infrastructure plays a central role in ensuring the reliable and efficient delivery of electricity across large distances. Given the significant capital requirements and complex engineering involved, regulators increasingly rely on competitive auctions to allocate the development and operation rights for new transmission projects. Auctions aim to promote cost efficiency, transparency, and timely investment in critical infrastructure.

Recent years have witnessed a growing trend towards bundling different types of transmission works — specifically, combining new projects and expansions of existing infrastructure into a single indivisible package for auction. A good example is the Chilean electricity transmission sector, in which since 2019 the National Energy Commission (CNE) and the system operator have adopted bundling strategies to enhance the attractiveness of expansion works that might not be appealing on their own. By forcing bidders to consider the entire package, bundling seeks to leverage the greater appeal of new works to promote investment in expansions critical for system reliability.

However, bundling heterogeneous infrastructure works raises significant challenges for auction design. New and expansion works typically differ in terms of risk, financing requirements, operational responsibilities, and payment structures. Moreover, firms' financing conditions – such as their cost of capital, access to debt markets, and internal risk assessments – are at least privately known and vary considerably across bidders. These features can introduce informational asymmetries that distort bidding behavior, affect allocative efficiency, and potentially create unintended rents for firms with advantageous financing capabilities.

This paper addresses the following research question: how should procurement auctions be optimally designed when bundling new and expansion transmission infrastructure works, taking into account private information on financing costs? We develop a theoretical framework in which bidders observe noisy signals about common construction costs and possess private information about their individual financing conditions. Bidders jointly submit offers for the bundled projects, and the regulator awards the contract based on the lowest total bid.

Our analysis shows that informational rents related to financing advantages persist even under optimal bidding, affecting the expected profits and participation incentives of firms. We further explore how regulatory restrictions on minimum bids, and the degree of financing cost observability influence the auction outcome and its efficiency.

While there is an extensive literature on auction theory, procurement mechanisms, and bundling strategies, to the best of our knowledge no prior work has systematically analyzed how bundling new and expansion infrastructure works interacts with private financing information to shape bidding behavior. By filling this gap, our study

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provides practical insights for regulators seeking to design more effective transmission auctions, especially in markets where financial heterogeneity among participants is significant.

Our theoretical analysis is motivated by recent auction reforms in Chile's transmission sector, where bundled procurement has been used to address financing constraints and overcome low bidder participation in expansion-only projects. We document a detailed case study from the 2021 international auction 229-185, in which the winning bidder had to price both new and conditional expansion works jointly under asymmetric financial conditions. This example illustrates the empirical relevance of our assumptions and connects directly to the mechanism design problem studied in the paper.

The remainder of the paper is structured as follows. Section 2 reviews the related literature. Section 3 provides brief context for understanding the Chilean electricity market and presents a case study of the international transmission auction of 2021 as motivation for our research. In Section 4, the theoretical model is presented. Section 5 discusses several extensions and robustness checks. Section 6 elaborates on policy implications and practical considerations. Section 7 concludes by summarizing the main findings and highlighting avenues for future research.

## 2. Literature review

We now present a review of the relevant literature, focusing on studies addressing commodity bundling, optimal auction design, procurement bidding mechanisms, and their implications for efficiency and profitability in various contexts, including power transmission infrastructure.

We identify several distinct strands within this literature.

First, classical works on commodity bundling and market efficiency include Adams and Yellen (1976), Palfrey (1983), and McAfee et al. (1989). Adams and Yellen (1976) introduced the distinction between pure and mixed bundling and emphasized bundling's role in extracting consumer surplus. Palfrey (1983) analyzed bundling's impact on buyer welfare under uncertain demand conditions, while McAfee et al. (1989) provided sufficient conditions under which bundling emerges as a profit-maximizing strategy. While these seminal contributions illuminate the general advantages of bundling, they largely focus on consumer markets and monopolistic sellers, rather than on competitive procurement settings with financing asymmetries. Our paper extends the bundling logic to a procurement context in which financing costs are privately observed by bidders, a dimension absent in the classical literature.

Second, the literature on optimal auction design and price discrimination offers critical insights into the structure of efficient mechanisms. Bulow and Roberts (1989) linked optimal auctions to price discrimination strategies, while Avery and Hendershott (2000) studied the auctioning of multiple products to heterogeneous buyers. These studies underline the importance of designing auctions that account for bidder asymmetries. However, neither explicitly models the case where bidders' financing conditions differ and are privately known, as we do. Our work builds on these ideas by analyzing how financing asymmetries impact auction outcomes under bundled procurement.

Third, researchers such as Armstrong (2000) and Maskin and Riley (2000) explored the comparative performance of different auction formats. Armstrong (2000) examined optimal auctions of multiple objects, considering bidder asymmetries, while Maskin and Riley (2000) revisited the revenue equivalence theorem under asymmetric bidder valuations. These contributions focus on auction formats but do not explicitly address how financing costs interact with auction design. Our model complements this literature by incorporating financing frictions into sealed-bid auctions for bundled infrastructure works.

Fourth, several studies focus directly on procurement auctions and bundling strategies. Chakraborty (1999) analyzed bundling decisions prior to selling multiple objects. Asker and Cantillon (2010) characterized optimal procurement mechanisms with private cost information. Li et al. (2015) investigated bundling decisions in sequential procurement tasks. Menicucci et al. (2015) demonstrated the optimality of pure bundling under certain conditions. More recently, Suzuki (2018, 2021) empirically studied the effects of bundling contracts in electricity procurement auctions in Japan, finding that bundling affects participation and bidding behavior. These studies advance the understanding of bundling in procurement but do not consider the role of private financing costs in shaping bidder strategies. Our paper explicitly models financing asymmetries as a critical determinant of bundling outcomes in procurement auctions.

Fifth, recent developments in auction and procurement theory emphasize the importance of bidder heterogeneity and information asymmetries. Aryal et al. (2023) provides an empirical approach for procurements with heterogeneous bidders, highlighting how cost and risk-aversion asymmetries affect bidding behavior and auction outcomes. Huang et al. (2022) examine bundling decisions in online auctions, investigating how auction parameters influence seller profits and buyer utility. These more recent studies stress the importance of heterogeneity and strategic interactions, but neither explicitly models private financing costs in an infrastructure procurement context with bundled works. Our paper complements and extends these insights by analyzing how privately known financing conditions affect bundled procurement outcomes in energy infrastructure auctions.

Sixth, there is a related literature on strategic procurement mechanisms and efficiency trade-offs. Chakraborty et al. (2021) studied procurement auctions with moral hazard, arguing that sometimes inefficient allocations can mitigate ex-post risk. While sharing a concern with efficiency under asymmetry, our focus differs: we analyze potential inefficiencies arising from adverse selection due to private financing information, not moral hazard.

Finally, dynamic bundling decisions have been studied by Lu and Zhao (2023), who analyzed intertemporal bundling in dynamic selling mechanisms. While related to our bundling framework, their focus is on time-based bundling in monopolistic contexts rather than competitive procurement of infrastructure.

Overall, while prior literature has made substantial contributions to bundling strategies, auction theory, and procurement design, few studies explicitly consider bundling under private financing asymmetries. Our paper addresses this gap by developing a procurement auction model where bidders face common-value uncertainty in construction costs and private-value asymmetries in financing conditions, with a specific application to the bundling of transmission infrastructure projects.

To summarize the relationship between previous work and our contribution, we provide a comparison in Table 1.

This paper thus contributes to the literature by formally modeling how bundling infrastructure works with different development, operational, and financing characteristics affects bidding strategies and auction outcomes, particularly when bidders have private information about financing costs — a feature that has received little attention in the auction theory or procurement literature to date.

## 3. Motivation

We now present a case study of Chile's 2021 international transmission auction which illustrates how real-world procurement authorities bundle works to overcome financing barriers, motivating the assumptions and policy relevance of our model.

**Table 1**
Comparison of prior literature and our contribution.

| Study | Setting | Bundling? | Financing costs modeled? | Private information modeled? |
|---|---|---|---|---|
| Adams and Yellen (1976) | Monopoly sales | Yes | No | No |
| Bulow and Roberts (1989) | Auctions (general) | No | No | Yes (cost heterogeneity) |
| Armstrong (2000) | Multiple object auctions | No | No | Yes (valuation asymmetry) |
| Asker and Cantillon (2010) | Procurement auctions | No | No | Yes (cost information) |
| Suzuki (2021) | Electricity procurement (empirical) | Yes | No | No |
| Chakraborty et al. (2021) | Procurement with moral hazard | No | No | Yes (moral hazard) |
| Aryal et al. (2023) | Procurement auctions | No | Yes | Yes (cost information) |
| **This paper** | Bundled infrastructure procurement | Yes | Yes (financing asymmetries) | Yes (financing and construction costs) |

### 3.1. Chilean electricity market

The Chilean electricity system is divided into 3 segments: generation, transmission and distribution. The transmission segment breaks into a National Transmission System (SEN), a Zonal Transmission System, a dedicated transmission system and a transmission system for development clusters.[1] The National Transmission System was created in 2017, with the merger of the northern system (*Sistema Interconectado del Norte Grande, SINC*) and the central and southern system (*Sistema Interconectado Central, SIC*). Therefore, it now represents a unified system that reaches almost the whole national territory, with a length of 3.100 km that allows it to cover over 98% of the national population by 2025.[2]

Even though the generation market has free entrance, there are vertical integration boundaries for companies participating in both generation and transmission segments. Hence, a company that participates in the generation segment cannot take part of the National Transmission System if the company individually holds more than 8% of its total investment value or there is a joined participation that reaches the threshold of 40% of the total investment value of the National Transmission System.[3]

The transmission segment has features of a natural monopoly, therefore, its expansion is regulated by law. Each year, the National Energy Commission (CNE) prepares a Technical Report for the Annual Transmission Expansion Plan, which establishes a priorizerd list of works to be added to the National Transmission System (SEN) and the Zonal System. It also establishes their basic technical characteristics, including estimates of: (1) the necessary Investment Value (I.V.); (2) its annual operation, maintenance and administration costs (O,M&A costs); (3) the expected useful life of each project; and (4) the budgeted construction period for the referential work of the project.

These power transmission infrastructure works may be new, which implies that they involve infrastructure that does not exist in any form at the time the requirement is determined, and there can also be expansion works, which consists of modifying, expanding or improving the capacity of transmission facilities that are already in operation and therefore have an identifiable owner. In the first case, the bidder to whom the execution is assigned must also operate the works during their useful life; in the second case, the expansion works are developed and constructed by the bidder who is assigned them, but their operation and ownership is typically handed over to the original owner of the work, who must pay the developer the amount stipulated by the latter in its economic proposal, operate it, and maintain it for the foreseeable future.

Once approved by the National Energy Commission (CNE), the works are included in a Decree on New and Expansion Works issued by the Ministry of Energy, starting a 12-month window where the works must be tendered. Since 2019, the awarding of new works is bundled to the awarding of expansion works, so that a package of works can only be tendered as a whole, indissolubly and unconditionally. The allocation principle implies that bundled works are awarded to the bidder with the lowest value for the whole package.

#### 3.1.1. Case study

The Annual Transmission Expansion Plan for 2020[4] included 47 works, reaching a total investment of USD 494 million. The Plan accounted for 14 projects for the National Transmission System, including 2 new works and 12 expansion works, adding up to nearly USD 301 million of the total investment. The rest of the works belonged to the Zonal System, including 7 new works and 26 expansion works, accounting for the remaining USD 193 million.[5]

In this context, the procurement process of a Bundled Works Group No. 1 took place, where the awarding of one of the new works for the National Transmission System was bundled to the awarding of two expansion works. The bundle had a total referential investment value of USD 200.784.815 and a referential annual operation, maintenance and administration cost of USD 3.211.981 (Details in Table 2). A 48-month window was granted in which the works must be built and operating.

The opening of economic offers included six bidders' proposals for this bundled package, each of which is detailed in Table 3.

Despite presenting the lowest global value for the group of works No. 1, the economic offer from company A was disqualified by the Coordinator, appealing to the fact that the proposal did not fulfill the bidding bases. More specifically, the Coordinator considered that the 1 USD investment value assigned to each of the expansion works did not internalize all the execution cost, breaching numeral 8.5 of

[1] Monitoring Report on Competition in the Electricity Market 2024 (Coordinador Eléctrico Nacional, 2025).

[2] See: https://www.coordinador.cl/sistema-electrico/.

[3] Monitoring Report on Competition in the Electricity Market 2024 (Coordinador Eléctrico Nacional, 2025).

[4] International Public Tender for New Works established by Exempt Decree No. 229/2021 and for Conditional Expansion Works established by Exempt Decree No. 185/2021 of the Ministry of Energy, Chile.

[5] Final Technical Report - Annual Transmission Expansion Plan for 2020 (Comisión Nacional de Energía, 2021).

**Table 2**
Bundled works group No. 1.
*Source:* Exempt Decree No. 229/2021 & Exempt Decree No. 185/2021 of the Ministry of Energy, Chile.

| Works | Referential investment value (USD) | Referential annual O, M & A costs (USD) | Useful life (years) |
|---|---|---|---|
| New Work: New S/E Switching Station, *Nueva Lagunas* and New 2 × 500 kV Line, *Nueva Lagunas* - *Kimal* | 194,462,361 | 3,111,398 | – |
| Expansion Work No. 1: Capacity Increase 2 × 220 kV *Tarapacá* - *Lagunas* Line, *Nueva Lagunas* - *Lagunas* Section | 3,790,073 | 60,641 | 47 |
| Expansion Work No. 2: Expansion of the *Kimal* 500 kV substation (IM) | 2,496,381 | 39,942 | 49 |

**Table 3**
Economic offers presented for bundled works Group No. 1.
*Source:* Minutes of the Opening of Financial Proposals and Minutes of the Evaluation of Financial Proposals issued by the National Electrical Coordinator (2022).

| Bidder | (1) | | | (2) | (3) | Global |
|---|---|---|---|---|---|---|
| | AVI (USD) | COMA (USD) | VATT (USD) | VI (USD) | VI (USD) | Value (USD) |
| Company A | 11,711,536.0 | 1,406,811.0 | 13,118,347.0 | 1.0 | 1.0 | 13,118,347.18 |
| Company B | 13,458,000.0 | 1,200,000.0 | 14,658,000.0 | 3,980,000.0 | 2,621,000.0 | 15,281,087.70 |
| Company C | 15,363,801.6 | 1,067,537.6 | 16,431,339.3 | 3,349,920.9 | 1,765,783.2 | 16,914,225.52 |
| Company D | 13,431,085.2 | 3,111,398.0 | 16,542,483.2 | 1,386,900.0 | 4,480,288.0 | 17,096,304.24 |
| Company E | 12,080,000.0 | 4,696,000.0 | 16,776,000.0 | 4,032,979.0 | 2,656,374.0 | 17,407,427.60 |
| Company F | 27,191,000.0 | 2,135,000.0 | 29,326,000.0 | 6,619,735.0 | 7,002,222.0 | 30,611,816.37 |

(1) New S/E Switching Station, *Nueva Lagunas* and New 2 × 500 kV Line, *Nueva Lagunas* - *Kimal*.
(2) Capacity Increase 2 × 220 kV *Tarapacá* - *Lagunas* Line, *Nueva Lagunas* - *Lagunas* Section.
(3) Expansion of the *Kimal* 500 kV substation (IM).

the administrative general bases. It was also pointed out that this investment value interfered with the calculation of the contract price, which was detrimental to both the owner of the works to be expanded and the general transmission system, as their remunerations are tied to the investment price.[6]

The economic offer from company F was also disqualified by the Coordinator, since its global value surpassed the Reserve Marginal Value (USD 27,037,991) and the Maximum Value (USD 25,750,468) established by the Coordinator for this group of works, breaching numerals 10.40.1 and 10.4.2 of the bases.[7] Therefore, the Bundled Works Group No. 1 was awarded to Company B who, after disqualification of Company A, became the bidder with the lowest global value for this package.[8]

The institutional features discussed above – particularly the bundling of new and expansion works and the limited interest in isolated upgrades – motivate our formal model that explicitly captures the implications of procurement design under financing heterogeneity. In particular, the frequent observation that expansion works are less attractive to bidders, as noted by the National Energy Commission in multiple Monitoring Reports,[9] supports the theoretical distinction in our model between "more bankable" and "less bankable" projects. Similarly, the decision to bundle multiple components into a single concession package raises the question of how different cost structures and financing conditions interact in determining bidding strategies and auction outcomes.

There are several reasons for the difference of attractiveness between new and expansion works. First, expansion works often involve greater technical and operational complexities, as they must be executed on existing infrastructure, increasing the risks of interferences and the need for coordination with facilities already in operation. Second, they may face more stringent environmental, social, and permitting constraints, since they intervene in areas with established easements or more organized local opposition. Finally, the fragmented nature of some expansions, involving short or dispersed segments, reduces economies of scale and makes it harder to achieve profitability levels comparable to larger new projects.

In addition, as noted by the National Energy Commission (CNE): expansion projects are often undertaken by smaller companies with more limited financial capacity, which accentuates the financing cost asymmetries relative to larger, more established firms typically involved in greenfield developments (Chapter 4, CEN Monitoring Report, 2021).

This diagnosis aligns closely with the rationale of our paper, as it highlights that, under financing cost asymmetries, a tender design bundling new works with expansions can help mitigate the lack of interest in the latter. By leveraging the revenues and economies of scale from new projects, such a design can offset the higher risks, lower profitability, and financing disadvantages associated with expansions.

To analyze these issues rigorously, in what follows, we construct a model of a procurement auction in which a regulator offers a bundle of works to competing firms. Each firm has private information about its own financing cost, which affects the total cost of delivering the bundle. This cost asymmetry reflects real-world differences in access to capital, credit terms, and internal project evaluation, as noted in Chilean transmission auctions. The model allows us to study how informational rents arise when bidders internalize the bundling and financing dimensions of the contract, and how equilibrium bidding behavior varies depending on the composition of the bundle and the distribution of financing types in the market.

[6] Minutes of the Evaluation of Financial Proposals issued by the National Electrical Coordinator (2022).
[7] Minutes of the Opening of Financial Proposals issued by the National Electrical Coordinator (2022).
[8] Minutes of tender award issued by the National Electrical Coordinator (2022).
[9] See, for example, Monitoring Report on Competition in the Electricity Market 2021 (Coordinador Eléctrico Nacional, 2021a).

By formalizing these empirical regularities in a tractable model, we aim to provide theoretical insights that are directly applicable to the design of procurement mechanisms in infrastructure sectors, particularly when financial frictions and regulatory design interact. The stylized setting of our model abstracts from many complexities of real-world contracts but captures their essential trade-offs — notably, how bundling affects allocative efficiency and the role of private financing information in shaping equilibrium bids.

## 4. The model

We consider a bidding process in which the regulator assigns the execution of two works $O^1$ and $O^2$ in a single and indivisible manner to one bidder.

The works will be assigned to the bidder that demands the lowest payment $b$ for the construction of both works and the operation of work 2 (new work, as opposed to work 1, which is an expansion work whose operation will not be the responsibility of the successful bidder but of the owner of the existing infrastructure to which the $O^1$ will add). Although the allocation will be for the lower value of payment $b$, this is broken down into two parts, a one-time payment against delivery for the work $O^1$ and an annuity for a total of $T$ periods to cover the construction and operation of the new work $O^2$, that is:

$$b = b^1 + b^2 \tag{1}$$
$$b^1 = \frac{B^1}{1+r}$$
$$b^2 = \sum_{t=1}^{T} \frac{B^2}{(1+r)^t}$$

obviously, it is irrelevant if the contract is awarded to the one who demands a lower payment $b$ or a lower amount $B = B^1 + B^2$ since the relationship between the two is monotonic.

The works are complementary in the sense that it is not possible to operate the work $O^2$ and, therefore, generate revenue from it, without first completing the work $O^1$.[10]

We will assume that the cost of construction and operation of the works (as a whole) is a value $v$ common to all bidders but unknown to them, who only observe an unbiased signal $v_i$.

In addition, each participant $i = 1, \ldots, n$ $(n \geq 2)$ faces a cost $c_i$ of financing the loans. This cost is specific to each participant and depends on its debt rationality and on the bidder's ability to collateralize the flow of payments it expects from the new work.

In our model, we assume that each firm's financing cost is privately known. While certain elements of firms' financial conditions – such as credit ratings, audited financial reports, or typical bond spreads – are publicly observable, the actual financing costs faced by a firm when executing a specific infrastructure project remain partially private. These financing costs depend not only on public indicators, but also on internal factors such as the firm's internal cost of capital, privately negotiated bank loans, bespoke debt covenants, project-specific risk assessments, and access to specialized financing instruments. Particularly in infrastructure markets in emerging economies, regulatory uncertainty, fluctuating sovereign risk, and firm-specific financial strategies further contribute to making effective financing costs imperfectly observable by external parties.

Thus, it is reasonable to model financing costs as containing a significant private component. Even if some partial observability exists, our key results would be robust: partial public information would reduce but not eliminate informational rents, as bidders would still retain private insights into their true financing conditions. Extending the model to explicitly incorporate partial observability would involve allowing financing cost signals to have both a public and private component, which would moderate but not fundamentally alter the strategic bidding behavior analyzed herein.

We will assume that both the signal $v_i$ and the cost $c_i$ come from known distributions with densities $f_v(\cdot)$ and $f_c(\cdot)$ and supports $[v_L, v_H]$ y $[c_L, c_H]$ respectively, with $v_L < v_H$ and $c_L < c_H$. Both distributions are assumed to be log-concave[11] and the signals are assumed to be independent of each other and independent and identically distributed among participants.

Finally, we will assume that the signals on construction and operating costs $v_i$ are averaged to the true value[12] (which implies that the quality of the information received by each proposer is the same), that is:

$$v = \frac{1}{n}\sum_{j=1}^{n} v_j$$

If a bidder $i$, who receives a cost signal $c_i$ and submits a bid for $b_i$ is assigned the bidding, construction and operation will receive profits for:

$$\pi_i(b_i) = b_i - (v + c_i)$$

since the payoff (and, therefore the strategy) of each proposer depends on two distinct random variables we might face the problem that there is no natural way to order two signals in two distinct dimensions,[13] however, in this case, an aggregate statistic is an obvious alternative for such ordering. Indeed, note that:

$$\begin{aligned}\pi_i(b_i) &= b_i - \left(\frac{1}{n}\sum_{j=1}^{n} v_i + c_i\right)\\ &= b_i - \left(\frac{1}{n}v_i + c_i + \frac{1}{n}\sum_{j\neq i} v_j\right)\\ &= b_i - \left(s_i + \frac{1}{n}\sum_{j\neq i} v_j\right)\end{aligned}$$

we will refer to the random variable $s_i$ as the "type" of bidder $i$ and note that it has a known distribution that depends on $f_c$ and $f_v$ will be log-concave with support on $[v_L + c_L, v_H + c_H]$.

### 4.1. Equilibrium of sealed envelope and first price auction

In the following we characterize a *Bayesian Nash Equilibrium* for the game in which the different bidders observe their type $s_{j\neq i}$ and decide, based on this, a bid according to a function $b(\cdot)$. The regulator observes all bids and assigns the bid to the bidder who demands the lowest payment $b(\cdot)$.

Without loss of generality, we focus on the strategy of bidder 1. We will assume that the bidder observes her type $s_1$ and submits a bid $b_1$. Conditional on her type and on her bid being the lowest of all (otherwise she does not win the bid) her expected utility is:

$$\pi_1^e(b_1) = \mathrm{E}\left[b_1 - s_1 - \frac{1}{n}\sum_{j=2}^{n} v_j \,\middle|\, s_1, b_1 \leq b_j \forall j \neq 1\right] \Pr\left[b_1 \leq b_j \forall j \neq 1\right]$$

[10] This eliminates the possibility of bidding for both works with the intention of abandoning $O^1$ or trying to renegotiate the associated payment later on.

[11] That is, the logarithm of each density is concave, i.e.: $\ln f(\theta x + (1-\theta)y) \geq \theta \ln f(x) + (1-\theta)\ln f(y)$. Most of the usual continuous distributions (e.g. normal, beta, gamma, exponential, uniform) satisfy this condition.

[12] This specification is commonly used in the common value bidding literature such as: Mares and Harstad (2003), De Silva et al. (2008), Goeree and Offerman (2003), or Menezes and Monteiro (2003) another alternative is to use the more classical Wilson (1968) specification in which $v$ distributes according to some known distribution $F_v$ and participants receive an unbiased signal $z$ from a distribution $F_{Z|V=v}$. Qualitatively, both specifications generate the same results, but the one used here simplifies the calculations considerably.

[13] See, for example, the discussion in Goeree and Offerman (2003).

$$\pi_1^e(b_1) = \left( b_1 - s_1 - \frac{1}{n}\sum_{j=2}^{n} \mathrm{E}[v_j | s_1, b_1 \le b_j \forall j \ne 1] \right) \Pr\left[ b_1 \le b_j \forall j \ne 1 \right] \quad (2)$$

Now, assuming that each of the other $n-1$ participants decides her bid according to a function $b(s_j)$ we note that this function must be strictly increasing[14] and, given that the support of $s_j$ is bounded, then so must be the path of $b(\cdot)$, that is, $b \in [b_L, b_H]$ for some values $b_L < b_H$.

Given this, it is natural to assume that $b_1$ must also belong to this interval since, if bidder 1 chooses $b_1 > b_H$ she will never win the bidding and, if $b_1 < b_L$ participant 1 will always win the bidding, but she will also win if $b_1 = b_L$ and will have higher profits.

Then, we can safely assume that there exists some type $\omega$ for which $b(\omega) = b_1$. With this we transform the game in which proposer 1 chooses an offer $b_1$ to one in which player 1 chooses which type to declare $\omega$ and the system automatically chooses what her bid will be based on the function $b(\cdot)$. The idea is, then, to implement bidding as a direct mechanism.[15]

**Proposition 1.** *The optimal bidding strategy is a solution of the following differential equation:*

$$\left\{ \frac{d}{d\omega}\left( b(\omega) - \frac{n-1}{n} E[v | s \ge \omega] \right) \right\} \frac{[1 - F_s(\omega)]}{(n-1) f_s(\omega)} - \left( b(\omega) = \frac{n-1}{n} E[v | s \ge \omega] - s_1 \right) = 0 \quad (3)$$

*where* $s = \min_j s_j$ *and* $F_s(\cdot)$ *is the cumulative distribution function of* $s$.

**Proof of Proposition 1.** See Appendix.

The right hand side of Eq. (3) corresponds to the marginal cost of marginally increasing $\omega$ that is, risking not winning the auction in spite of having the lower cost type, corresponds to the marginal benefit of increasing the bid (a higher profit) conditional on still winning the auction. In other words, Proposition 1 shows that the bidder chooses her bid to balance two forces: the higher she bids, the lower the probability of winning the auction, but the higher the profit she gets if she wins. The bidder internalizes both its private financing condition and the common uncertainty about construction costs when setting her optimal bid.

From Proposition 1 and following the *Revelation Principle*[16] it is possible to find the *Bayesian Nash Equilibrium* of this game with a mechanism that is "direct" (each proposer simply chooses a type to declare) and incentive compatible (the revealed type is the one that each proposer actually has). This means that we can concentrate exclusively on the equilibrium in which $\omega = s_1$, thus:

$$\left\{ \frac{d}{ds_1}\left( b(s_1) - \frac{n-1}{n} \mathrm{E}[v | s \ge s_1] \right) \right\} \frac{[1 - F_s(s_1)]}{(n-1) f_s(s_1)} = \left( b(s_1) - \frac{n-1}{n} \mathrm{E}[v | s \ge s_1] - s_1 \right) \quad (4)$$

Next we propose the following result:

**Proposition 2.** *The symmetric Bayesian Nash Equilibrium of the game is such that all participants bid* $b_i$ *using the same strategy* $b(\cdot)$ *given by the following equation:*

$$b_i = b(s_i) = \frac{n-1}{n} E[v | s \ge s_i] + E[y_i | y_i \ge s_i] \quad (5)$$

*where* $y_i = \min_{j \ne i} s_j$.

[14] In this regard, it is sufficient to note that the signals of the different participants are affiliated, so that observing a higher value of $v_1$ makes it more likely that $v$ will be higher for all participants, making it reasonable to request a higher payment when receiving a higher type $s_1$.

[15] That is, it "asks" each proposer directly what her type is. Note, however, that nothing prevents the proposer from declaring a type other than her own.

[16] See, for example, Fudenbverg and Tirole (1995).

**Proof of Proposition 2.** See Appendix.

Proposition 2 shows that the optimal bid for each participant equals the expected common construction cost, adjusted downward for idiosyncratic financing advantages. Specifically, bidders incorporate the advantage they have over the second-lowest bidder in terms of financing costs, which determines the expected informational rent they can extract. Thus, the equilibrium bid reflects both common project cost estimates and private financing advantages.

Given the result put forward inProposition 2 we can now obtain the expected profit of all participants.

**Corollary 1.** *A participant's expected profits when bidding accordingly to* $b(s_i)$ *are:*

$$\pi_i^e(s_i) = \left( E[y_i | y_i \ge s_i] - s_i \right) \left[ 1 - F_s(s_i) \right]^{n-1} \quad (6)$$

**Proof of Corollary 1.** See Appendix.

Corollary 1 implies that (reasonably) firm $i$'s profit expectation is given by its expected cost advantage (with respect to the second competitor with the lowest expected cost advantage) times the probability that its cost signal is the lowest of all participants.

Corollary 1 also indicates the important result that the bidding mechanism is such that the works bundle is allocated efficiently, i.e., it will be developed by the bidder that expects to be able to do it at the lowest total cost.

**Corollary 2.** *Eq. (5) can be rewritten as:*

$$b_i = E\left[ V + c | \, s \ge s_i \right] + E[y_1 - s_i | y_1 \ge s_i] \quad (7)$$

**Proof of Corollary 2.** See Appendix.

The first term of this expression corresponds to the expected value of the development, operation and financing costs, given the signals received by the participant, and the second term corresponds to the expected value of its informational rent and lower private costs (financing). This facilitates the economic interpretation of the bidding strategy.

### 4.2. Comparative statics

Based on the result presented inProposition 2, we analyze what happens to a participant's optimal proposal and expected profits when the cost signal he receives increases (i.e., when he expects the execution and operation of the works to become more costly).

We base our analysis on the following result:

**Proposition 3.** *The effect of a change in the cost signal received by a firm on the equilibrium expected profit from winning the project is negative or equal to zero, that is:*

$$\frac{\partial \pi^e(s_i)}{\partial s_i} \le 0$$

**Proof of Proposition 3.** See Appendix.

Proposition 3 shows that, ceteris paribus, when a firm receives a higher cost signal, that is when it expects the cost of developing the project to be higher, its expected profit from winning the project decreases. The rationale behind this result comes from the double effect: there is a lower margin but, in addition to the higher cost of developing the project, there is a lower profit margin. Consequently, the probability of winning the bid decreases, as it is likely that another participant will have a lower cost signal, or will be able to finance the project at a lower cost.

## 5. Extensions to the model

In this section we examine some extensions to the basic model that allow us to explore the behavior observed in the specific 2021 auction that motivated this research.

### 5.1. Extension 1: Partial endogeneity in financing costs

We minimally extend the model to assume that firms have some control over $c_i$ the private cost of financing the project as follows: instead of receiving a $c_i$ signal of the total cost of financing the project, we assume that firms each receive interest rate signals $r^1$ and $r^2$ associated with the part of the cost that they can finance with a collateralizable future income stream (associated with the new work, $r^2$) and another short-term rate ($r^1$) associated with the part of the cost of the expansion work that must be financed for a period and that will not generate a collateralizable payment stream. We naturally assume that $r^1 > r^2$.

In addition, each firm receives a signal of the limit $0 < \lambda \leq 1$ for the total cost it can finance against the flow it can offer as collateral. These signals depend, among other things, on the specific characteristics of the firm, like its access to financial markets and its reputation.

Given the above, the cost $c_i$ becomes:

$$c_i = \left[(1 + r^2)\delta + (1 + r^1)(1 - \delta)\right] V$$

where $\delta$ is the fraction of the infrastructure works that the firm decides to finance by offering as collateral the expected flow of payments. Obviously $\delta \leq \lambda$.

Now, given that:

$$s_i = \frac{1}{n} v_i + c_i$$

it follows that if $s_i$ is strictly increasing in $c_i$ and, given that $\pi^e(s_i)$ is decreasing in $s_i$, it is straightforward to note that the firm will want to minimize $c_i$ so the solution to its choice problem of $\delta$ will be, simply, to set $\delta = \lambda$.

Therefore, a company that may finance its investment primarily using future cash flows will choose to do so.

However, Eq. (1) shows that this is not exogenous, the firm can affect the value of future collateralizable flows if it allocates a larger portion of its bid collection to new construction (i.e., it decreases $b^1$ and feeds $b^2$) and, rationally, it should choose to do so since nothing in the bidding process structure prevents it from doing so.

Consequently, we can put forward the following result whose proof is contained in the analysis above:

**Proposition 4.** *A firm that observes a relatively high value of the signal on the limit for the total cost it can finance against the flow it can offer as collateral, will direct a larger portion of the payment to the future flows associated with the new work.*

**Proof of Proposition 4.** Follows from discussion above.

The rationale behind the result presented in Proposition 4 is that whenever a firm receives a high value of ($\lambda$), will devote a larger portion of the payment to the future flows associated with the new work, because this action does increase its chances of winning the bid and, at the same time, increases its expected benefits from doing so.

This results rationalizes the behavior observed in the 2021 auction in which one firm bade 1 dollar for both expansion works included in the bundle.

### 5.2. Extension 2: Minimum investment value for the expansion project

Finally, we explore the counterfactual effect that the auction rules could have if the bidding conditions considered a minimum investment value to be declared for the expansion work. This is precisely the scenario that would arise from the regulator's interpretation of the obligation of bidders to truly report the costs of expansion works and not only that of the bundle.[17]

Recall that our assumption, according to the design of the auction under study, is that each bidder demands a full payment for the work package according to (1) which is not subject to any minimum value restrictions.

By imposing a minimum investment value for the expansion work, say $\bar{b}_1$, so that $b_1 \geq \bar{b}_1$ we have to consider several impact possibilities depending on what are the optimal values $b_1^*$ and $b_2^*$ that the company would declare in the absence of this constraint.

Indeed, it is easy to see that if $b_1^* \geq \bar{b}_1$ then the constraint has no effect on the bidder's optimal strategy and the bidder will demand the same payoff, with the same structure as if no minimum declared investment value is imposed.

The situation changes when $b_1^* < \bar{b}_1$ as this case forces to reduce the financing of the work package that is directed towards the new work which reduces the company's ability to arbitrage rates between the two and, ultimately, increases the costs of execution.

Consequently, the firm will choose $b_1^{**} = \bar{b}_1$ and $b_2^{**} < b_2^*$ as a position to build the works package but, as decreasing $b_2$ has an impact on the financing cost $c$ it is expected that $b_1^{**} + b_2^{**} > b_1^* + b_2^*$ since the constraint forces it to choose a combination of financing sources that is different from the one it would have chosen if it did not face the constraint of declaring a value of investment for the expansion work which is, at least, $\bar{b}_1$.

An interesting result is that the inclusion of a minimum investment requirement may break the efficiency of the auction process in the sense that it may result in assigning the execution of the works to a bidder who, in the absence of the requirement, would have faced a higher cost of financing the works. This would imply that the mandate to execute the works at the lowest possible cost would not be fulfilled.

Indeed, let us consider the following situation. In the absence of restrictions on the value $b_1$ the bidder $i$ would have submitted a proposal for the amount:

$$b_i^* = b_{1,i}^* + b_{2,i} *$$

while applicant $j$ would have submitted a proposal for:

$$b_j^* = b_{1,j}^* + b_{2,j} *$$

we assume that $\bar{b}_1 > b_{1,i}^*$ and $\bar{b}_1 > b_{1,j}^*$, and assume, without loss of generality, that $b_i^* > b_j^*$ so that the works would have been efficiently assigned to applicant $j$ before applicant $i$. Suppose, consistent with this result, that the financing costs are such that:

$$c_i(b_{1,i}^*, b_{2,i}^*) > c_j(b_{1,j}^*, b_{2,j}^*)$$
$$r_{1,i} b_{1,i}^* + r_{2,i} b_{2,i}^* > r_{1,j} b_{1,j}^* + r_{2,j} b_{2,j}^*$$

and suppose that firms' $j$ advantage comes mainly from having a lower cost of equity capital, so that $r_{2,j} < r_{2,i}$ but $r_{1,i} < r_{1,j}$[18] then, by imposing

[17] The reason to consider this normative change emerges from the previous result. Since bidding firms may have an incentive to demand a low payment for the expansion work this may affect the owner of the existing infrastructure who must pay, operate, and maintain the addition and is assigned a payment from the system that is proportional to the payment it made. This implies that the owner of the existing work may have to pay an unrealistically low price for the expansion at the cost of receiving a low payment for its operation and maintenance for some time (until the next regulatory process). In this way, the incentives of the bidder and those of the owner of the infrastructure to be expanded may not be aligned.

the constraint $b_1 \geq \bar{b}_1$ and assuming that it is active for both firms,[19] then it suffices that:

$$\frac{r_{2,i} b^{**}_{2,i} - r_{2,j} b^{**}_{2,j}}{r_{1,j} - r_{1,i}} \leq \bar{b}_1$$

to verify that:

$$r_{1,i} b^{**}_{1,i} + r_{2,i} b^{**}_{2,i} < r_{1,j} b^{**}_{1,j} + r_{2,j} b^{**}_{2,j}$$

This would allow firm $i$ to make a cheaper proposal than firm $j$ which would lead to the bidding result being reordered and now the works bundle is more likely to be assigned to bidder $i$ rather than bidder $j$ which would introduce and inefficiency and force a higher cost for the work than would have been the case without the restriction.

Hence, we can summarize this discussion in the following result:

**Proposition 5.** *Requiring as part of the bidding process a minimum value of the investment for one of the works may have the effect of, first, making the execution of the package of works more expensive (if it forces bidders to change the way they will finance them with respect to what would be optimal without the requirement) and, second, it may even lead to the works being allocated inefficiently.*

**Proof of Proposition 5.** Follows from discussion above.

It should be noted that this is not to say that a stated minimum investment value could not be required for the expansion work, but the regulator should be reasonably sure that the social benefit that this restriction placed on the efficient use of financing alternatives will bring in is at least large enough to offset the cost imposed on society by the possibly inefficient allocation of expansion works.

In either case, it is worth noticing that this problem emerges solely from the decision of the authority to remunerate the operation of the expansion works as a function of the winning bidder declaration of its costs. This restriction is, ultimately, unnecessary if the purpose of the auction is to allocate the development of the bundle of infrastructure at the minimum possible cost.

## 6. Implementation challenges

While the proposed auction framework offers a theoretically optimal design for bundling new and expansion infrastructure works, its practical implementation may encounter several challenges.

First, accurately verifying firms' true financing costs remains difficult. Although credit ratings, debt issuance terms, and general financial statements are publicly available, firms' effective project-specific financing conditions often rely on private negotiations and internal capital structures that are not fully observable to regulators. Designing mechanisms that require partial disclosure of financing arrangements without deterring participation would be a delicate task.

Second, regulators must be able to monitor and enforce the contractual obligations associated with the separate payment streams for different works. In practice, ensuring that payments are appropriately allocated between expansion and new works, and that cost declarations are not strategically distorted, requires robust auditing and reporting mechanisms.

Third, in settings where regulatory capacity is limited – particularly in emerging countries – auction design complexities may reduce transparency and increase administrative costs. Simplified auction formats that retain the core insights of the model while minimizing informational and operational burdens may be more practical in such environments.

Finally, regulators must anticipate strategic behavior by bidders. Firms may attempt to structure bids to exploit expected future regulatory revisions (such as adjustments to continuation payments or operating cost allowances) if these are imperfectly forecastable at the time of bidding.

In addition to the theoretical contributions, this paper provides a case study of an auction carried on in the Chilean electricity market, in which the auction structure, participation behavior, and disqualified bids provide empirical support for the interactions captured by our theoretical model. Future work could build on this analysis by incorporating richer institutional features, such as continuation payments or dynamic participation decisions, to better match the evolving procurement practices in emerging markets.

Despite these challenges, the insights provided by the model – particularly regarding the importance of recognizing financing asymmetries and carefully structuring bundled procurements – may prove useful. Acknowledging and preparing for these practical difficulties can help regulators better adapt theoretical auction principles to real-world procurement contexts.

## 7. Concluding remarks

This paper has analyzed the optimal bidding behavior in auctions for bundles of power transmission infrastructure works, considering the presence of private information regarding bidders' financing costs. We show that bundling new and expansion works, when properly designed, can enhance participation and achieve efficiency even under financing asymmetries. Our theoretical framework offers novel insights into the design of procurement mechanisms where cost heterogeneity and information asymmetries are significant factors.

These insights are directly motivated by procurement practices observed in infrastructure sectors. In Chile, for instance, the National Energy Commission (CNE) and the National Electrical Coordinator have, since 2019, implemented auction formats that bundle new transmission works with expansions of existing infrastructure into single indivisible packages. This bundling approach, formally specified in the Transmission Expansion Plans and auction guidelines (Coordinador Eléctrico Nacional, 2021b), was developed to overcome limited participation in stand-alone expansion projects, which often involve operational complexity and uncertain revenue flows. Our model's emphasis on bundling and financing asymmetries reflects these institutional challenges.

Similarly, Brazil's electricity regulator ANEEL has implemented bundled auctions where greenfield projects and expansions are auctioned jointly under unified concession contracts. For example, Transmission Auction 1/2021 required bidders to submit a single remuneration value that accounted for the financing structures and risk heterogeneity across works (ANEEL, 2021). In Japan, empirical research on electricity procurement (Suzuki, 2018, 2021) also documents the strategic consequences of bundling, though with a focus on cost and efficiency outcomes rather than financing asymmetries.

These real-world examples reinforce the practical importance of designing auctions that explicitly account for financing heterogeneity. They also illustrate the empirical relevance of the informational rents and bid distortions characterized in our theoretical model.

Lastly, Tinbergen Rule establishes that, to achieve several objectives the authority must have at least the same number of instruments. This golden rule also holds in this setup. If the regulator wants to assign the building of the required new transmission infrastructure to the bidder with the lowest overall development cost and, *at the same time*, wants to get a trustworthy signal of the cost of construction of the expansion works it will need a different instrument for each purpose. Trying to achieve both desired results with just the design of the auction rules will not work if there is some sort of heterogeneity between potential

[18] This does not necessarily imply that $b^*_{2,j} \geq b^*_{2,i}$ and $b^*_{1,j} < b^*_{1,i}$, although this is to be expected if companies can choose how to allocate financing from one work to the other.

[19] This should be the case since we assumed that both firms chose optimal values of $b_1$ below the minimum in the unconstrained equilibrium.

bidders that they can strategically exploit. This is what might have happened in the specific auction that motivated this theoretical work.

Nonetheless, several limitations of the model should be acknowledged. First, while we assume that financing costs are purely private information, in practice financing conditions may be partially observable through credit ratings, financial disclosures, or bond market behavior. Although we argue that a substantial private component remains relevant, modeling partial observability explicitly would offer a more nuanced analysis. Second, our model abstracts from financial market volatility and regulatory shocks, which could dynamically affect firms' financing conditions after bidding. Third, the auction framework considered is static and single-shot; in real-world settings, repeated auctions or staged procurement processes could allow bidders to update their strategies over time.

These limitations suggest promising directions for future research. Extending the model to a setting with partial public signals about financing costs, or to a dynamic auction environment where bidders face sequential procurement opportunities, could provide deeper insights into the strategic behavior of firms under regulatory uncertainty. Moreover, investigating how alternative regulatory interventions – such as financing condition disclosures, bid caps, or risk-sharing mechanisms – could mitigate financing asymmetries would have valuable policy implications.

Despite these simplifications, our analysis highlights the critical importance of recognizing financing asymmetries in bundled procurement auctions. By incorporating financing heterogeneity into the design of transmission infrastructure auctions, regulators can improve both the efficiency and competitiveness of procurement processes in complex infrastructure sectors.

## CRediT authorship contribution statement

**Valentina Apablaza:** Writing – original draft, Investigation, Conceptualization. **Andrés Hernando:** Writing – original draft, Investigation, Formal analysis, Conceptualization. **Mauricio G. Villena:** Writing – original draft, Investigation, Formal analysis, Conceptualization.

## Appendix

This appendix presents the proofs of the different propositions and corollaries of the model in the order in which they are enunciated in the text.

**Proof of Proposition 1.** Let $\omega$ be the type bidder 1 declares when her type is $s_1$, given that the optimal bidding function $b(\cdot)$ is increasing in its argument, the expected profits depend on the expected value of $v$ given that $b(\omega)$ is the lowest bid and the probability of $b(\omega)$ being the lowest bid:

$$\pi_1^e(\omega) = \left( b(\omega) - s_1 - \frac{1}{n}\sum_{j=2}^{n} \mathrm{E}[v_j|s_1, b(\omega) \le b(s_j)\forall j \ne 1] \right) \times \Pr\left[b(\omega) \le b(s_j)\forall j \ne 1\right]$$

given that $b(\cdot)$ is monotone, then $b(x_1) < b(x_2)$ if and only if $x_1 < x_2$, thus:

$$\pi_1^e(\omega) = \left( b(\omega) - s_1 - \frac{n-1}{n}\mathrm{E}[v|s_1, \omega \le s_j \forall j \ne 1] \right) \Pr\left[\omega \le s_j \forall j \ne 1\right]$$

Defining $s = \min_j s_j$ and the fact that the $n-1$ types of the other participants are i.i.d. to each other and letting $F_s(\cdot)$ be the cumulative distribution of $s$:

$$\begin{aligned} \pi_1^e(\omega) &= \left( b(\omega) - s_1 - \frac{n-1}{n}\mathrm{E}[v|\omega \le s] \right) \Pr\left[\omega \le s\right] \\ \pi_1^e(\omega) &= \left( b(\omega) - s_1 - \frac{n-1}{n}\mathrm{E}[v|\omega \le s] \right) \left[1 - F_s(\omega)\right]^{n-1} \end{aligned} \tag{8}$$

Now, we are able to differentiate this expression and impose the first order condition to obtain the optimal strategy of bidder 1 given that the other bidders are generating bids according to the function $b(\cdot)$.

$$\begin{aligned} \pi^{e_1\prime}(\omega) &= \left\{ \frac{d}{d\omega}\left( b(\omega) - \frac{n-1}{n}\mathrm{E}[v|s \ge \omega] \right) \right\} [1 - F_s(\omega)]^{n-1} \\ &\quad - (n-1)\left( b(\omega) - s_1 - \frac{n-1}{n}\mathrm{E}[v|\omega \le s] \right) \\ &\quad \times \left[1 - F_s(\omega)\right]^{n-2} f_s(\omega) = 0 \\ &= \left\{ \frac{d}{d\omega}\left( b(\omega) - \frac{n-1}{n}\mathrm{E}[v|s \ge \omega] \right) \right\} \frac{\left[1 - F_s(\omega)\right]}{(n-1) f_s(\omega)} \\ &\quad - \left( b(\omega) - \frac{n-1}{n}\mathrm{E}[v|s \ge \omega] - s_1 \right) = 0 \quad \blacksquare \end{aligned}$$

**Proof of Proposition 2.** Begin by defining $y_1 = \min_{j \ne 1} s_j$ and notice that $\frac{f_s(s_1)(n-1)}{1-F_s(s_1)} = \frac{f_{y_1}(s_1)}{1-F_{y_1}(s_1)}$. Using the following result from applying the Fundamental Theorem of Calculus to a conditional expectation on a truncated distribution:

$$\begin{aligned} \mathrm{E}[Y|X \ge x] &= \int_x^\infty \mathrm{E}[Y|X = t] \frac{f_X(t)}{1 - F_X(x)} dt \\ \frac{d}{dx}\mathrm{E}[Y|X \ge x] &= -\mathrm{E}[Y|X = x] \frac{f_X(x)}{1 - F_X(x)} \\ &\quad + \int_x^\infty \mathrm{E}[Y|X = t] \frac{f_X(t)}{(1 - F_X(x))^2} f_X(x) dt \\ &= \left(-\mathrm{E}[Y|X = x] + \mathrm{E}[Y|X \ge x]\right) \frac{f_X(x)}{1 - F_X(x)} \end{aligned} \tag{9}$$

we get:

$$s_1 = \mathrm{E}[y_1|y_1 = s_1]$$

Given that:

$$\begin{aligned} \frac{d}{ds_1}E[y_1|y_1 \ge s_1] &= \left[-\mathrm{E}[y_1|y_1 = s1] + \mathrm{E}[y_1|y_1 \ge s_1]\right] \frac{f_{y_1}(s_1)}{1 - F_{y_1}(s_1)} \\ \Rightarrow s_1 = \mathrm{E}[y_1|y_1 = s_1] &= -\frac{d}{ds_1}E[y_1|y_1 \ge s_1] \frac{1 - F_s(s_1)}{f_s(s_1)(n-1)} + \mathrm{E}[y_1|y_1 \ge s_1] \end{aligned}$$

then, replacing in (4):

$$\begin{aligned} &\left\{ \frac{d}{ds_1}\left( b(s_1) - \frac{n-1}{n}\mathrm{E}[v|s \ge s_1] \right) \right\} \frac{[1 - F_s(s_1)]}{(n-1) f_s(s_1)} \\ &- \left( b(s_1) - \frac{n-1}{n}\mathrm{E}[v|s \ge s_1] + \frac{d}{ds_1}E[y_1|y_1 \ge s_1] \frac{1 - F_s(s_1)}{f_s(s_1)(n-1)} \right. \\ &\qquad \left. -\mathrm{E}[y_1|y_1 \ge s_1] \right) = 0 \\ &\left\{ \frac{d}{ds_1}\left( b(s_1) - \frac{n-1}{n}\mathrm{E}[v|s \ge s_1] - E[y_1|y_1 \ge s_1] \right) \right\} \frac{[1 - F_s(s_1)]}{(n-1) f_s(s_1)} \\ &\qquad - \left( b(s_1) - \frac{n-1}{n}\mathrm{E}[v|s \ge s_1] - \mathrm{E}[y_1|y_1 \ge s_1] \right) = 0 \end{aligned}$$

defining:

$$\phi(s_1) = b(s_1) - \frac{n-1}{n}\mathrm{E}[v|s \ge s_1] - E[y_1|y_1 \ge s_1] \tag{10}$$

then the above equation may be rewritten as:

$$\phi'(s_1) - \phi(s_1)\frac{f_{y_1}(s_1)}{1 - F_{y_1}(s_1)} = 0$$

Defining:

$$P(s_1) = \exp\left( - \int_{\frac{1}{n}v_L + c_L}^{s_1} \frac{f_{y_1}(u)}{1 - F_{y_1}(u)} du \right)$$

Since, we know that:

$$\begin{aligned} P'(s_1) &= -\frac{f_{y_1}(s_1)}{1 - F_{y_1}(s_1)} \exp\left( - \int_{\frac{1}{n}v_L + c_L}^{s_1} \frac{f_{y_1}(u)}{1 - F_{y_1}(u)} du \right) \\ &= -\frac{f_{y_1}(u)}{1 - F_{y_1}(u)} P(s_1) \end{aligned}$$

We can obtain:

$$\begin{aligned}\phi'(s_1)P(s_1) - \phi(s_1)\frac{f_{y_1}(s_1)}{1-F_{y_1}(s_1)}P(s_1) &= 0 \\ \left[\phi(s_1)P(s_1)\right]' &= 0 \\ \phi(s_1)P(s_1) &= K\end{aligned} \tag{11}$$

Where $K$ is some constant.

Given that:

$$P\left(s_L = \frac{1}{n}v_L + c_L\right) = 1$$

replacing in (11) we have:

$$\phi'(s_L) - \phi(s_L)f_{y_1}(s_L) = 0$$

Noting that this equality can only hold if every term in the sum is null (and that $f_{y_1}(s_L) \neq 0$ by definition), we conclude that $K = 0$ and we have, then, that the optimal strategy of the proponent is given by:

$$b(s_1) = \frac{n-1}{n}\mathrm{E}[v|s \geq s_1] + \mathrm{E}[y_1|y_1 \geq s_1]$$

so

$$b(s_i) = \frac{n-1}{n}\mathrm{E}[v|s \geq s_i] + \mathrm{E}[y_i|y_i \geq s_i]$$

where $y_i$ is defined analogously to $y_1$ and the final results uses the fact that any bidder could have been labeled bidder 1. ■

**Proof of Corollary 1.** From Eq. (8) and the fact that optimally $\omega = s_1$ follows that the expected profits por player $i$ are given by:

$$\pi_i^e = \left(b(s_i) - s_i - \frac{n-1}{n}\mathrm{E}[v|s \geq s_i]\right)\left[1 - F_s(s_i)\right]^{n-1}$$

replacing the known form of $b(\cdot)$:

$$\begin{aligned}\pi_i^e &= \left(\frac{n-1}{n}\mathrm{E}[v|s \geq s_i] + \mathrm{E}[y_1|y_1 \geq s_i] - s_i - \frac{n-1}{n}\mathrm{E}[v|s \geq s_1]\right) \\ &\quad \times \left[1 - F_s(s_i)\right]^{n-1} \\ &= \left(\mathrm{E}[y_1|y_1 \geq s_i] - s_i\right)\left[1 - F_s(s_i)\right]^{n-1}\end{aligned}$$ ■

**Proof of Corollary 2.** Beginning with Eq. (5):

$$\begin{aligned}b_i = b(s_i) &= \frac{n-1}{n}\mathrm{E}[v|s \geq s_i] + \mathrm{E}[y_1|y_1 \geq s_i] \\ &= \mathrm{E}\left[\frac{n-1}{n}v \middle| s \geq s_i\right] + \mathrm{E}[y_1|y_1 \geq s_i] \\ &= \mathrm{E}\left[\frac{1}{n}\sum_{j=1}^{n} v_j - \frac{1}{n}v_i \middle| s \geq s_i\right] + \mathrm{E}[y_1|y_1 \geq s_i] \\ &= \mathrm{E}\left[V - s_i + c_i \middle| s \geq s_i\right] + \mathrm{E}[y_1|y_1 \geq s_i] \\ &= \mathrm{E}\left[V + c \middle| s \geq s_i\right] - s_i + \mathrm{E}[y_1|y_1 \geq s_i] \\ &= \mathrm{E}\left[V + c \middle| s \geq s_i\right] + \mathrm{E}[y_1 - s_i|y_1 \geq s_i]\end{aligned}$$ ■

**Proof of Proposition 3.** First, we notice that:

$$\begin{aligned}b_i(s_i) &= \frac{n-1}{n}\mathrm{E}[v|s \geq s_i] + \mathrm{E}[y_1|y_1 \geq s_i] \\ \Rightarrow \frac{\partial b_i(s_i)}{\partial s_i} &= \frac{n-1}{n}\left(-\mathrm{E}[v|s = s_i] + \mathrm{E}[v|s \geq s_i]\right)\frac{f_s(s_i)}{1-F_s(s_i)} \\ &\quad + \left(-\mathrm{E}[y_1|y_1 = s_i] + \mathrm{E}[y_1|y_1 \geq s_i]\right)\frac{f_y(y_1)}{1-F_y(y_1)} \\ &= \frac{n-1}{n}\left(-\mathrm{E}[v|s = s_i] + \mathrm{E}[v|s \geq s_i]\right)\frac{f_s(s_i)}{1-F_s(s_i)} \\ &\quad + \left(-s_i + \mathrm{E}[y_1|y_1 \geq s_i]\right)\frac{f_y(y_1)}{1-F_y(y_1)}\end{aligned}$$

Now, random variables $(X_1, X_2, \ldots, X_n)$ with support in a set $D$ are said to be affiliated if the joint density $f : D \to \Re$ exhibits the monotonic multivariate likelihood ratio property.

That is, given two vectors $x$ and $y$ and defining the supremum and infimum by coordinates as follows:

$$\begin{aligned}x \vee y &= (\max\{x_1, y_1\}, \max\{x_2, y_2\}, \ldots, \max\{x_n, y_n\}) \\ x \wedge y &= (\min\{x_1, y_1\}, \min\{x_2, y_2\}, \ldots, \min\{x_n, y_n\})\end{aligned}$$

then:

$$f(x \vee y)f(x \wedge y) \geq f(x)f(y)$$

Intuitively, this means that, as the value of $X_i$ increases, it is more likely to observe higher realizations of the other $X_j$, $j \neq i$.

There are two important results that we will use here, the first is that if the variables $(X_1, X_2, \ldots, X_n)$ are independent of each other, then they are affiliated. The second is that if $u(x)$ is an increasing function of its arguments and the random variables $X$ are affiliated then: $\mathrm{E}[u(X)|x_k = a]$ is increasing in $a$.[20]

We can also show that $E[u(x)|X \geq z]$ is non-decreasing in $z$. Note that:

$$\begin{aligned}\mathrm{E}[u(x)|X \geq z_1] &= \int_{z_1}^{\infty} \mathrm{E}[u(x)|x = t]\frac{f_X(t)}{1-F_X(z_1)}dt \\ \mathrm{E}[u(x)|X \geq z_2] &= \int_{z_2}^{\infty} \mathrm{E}[u(x)|x = t]\frac{f_X(t)}{1-F_X(z_2)}dt \\ \Rightarrow E[u(x)|X \geq z_2] - E[u(x)|X \geq z_1] &= \int_{z_1}^{z_2} \mathrm{E}[u(x)|X = t]f_X(t) \\ &\quad \times \left[\frac{1}{1-F_X(z_2)} - \frac{1}{1-F_X(z_1)}\right]dt \\ &= \frac{F_X(z_2) - F_X(z_1)}{(1-F_X(z_2))(1-F_X(z_1))} \\ &\quad \times \int_{z_1}^{z_2} \mathrm{E}[u(x)|X = t]f_X(t)dt\end{aligned}$$

It is straightforward to see that, as long as $u(x)$ is non-decreasing and $z_2 \geq z_1$ then the latter expression will be non-negative from which it follows that $E[u(x)|Z \geq z]$ is non-decreasing in $z$.

Now, because of the affiliation between the random variables $s_j$ $j = 1, 2, \ldots, n$ and given that $v = n(s - c)$ and $y_i = \min_{j \neq i} s_j$ are (at least) non-decreasing functions of $s_i$ and that, moreover, $E[u(x)|X \geq z]$ is non-decreasing in $z$ then we have that $b_i(s_i)$ is non-decreasing in $s_i$. That is, all else kept constant, if a firm receives a larger $s_i$ signal (indicating that it expects the project to be more costly) then its optimal response is to demand a larger payment for its completion. In other words, the optimal posture of any bidding participant is non-decreasing in the cost signal it receives.

Now, we obtain the expected profit equation, based on (6):

$$\pi^e(s_i) = \left(\mathrm{E}[y_i|y_i \geq s_i] - s_i\right)\left[1 - F_s(s_i)\right]^{n-1} \tag{12}$$

Equation (12) allows us to differentiate the expected profit with respect to the cost signal, given by $s_i$, which produces:

$$\begin{aligned}\pi^e(s_i) &= \left(\mathrm{E}[y_i|y_i \geq s_i] - s_i\right)\left[1 - F_s(s_i)\right]^{n-1} \\ \Rightarrow \frac{\partial \pi^e(s_i)}{\partial s_i} &= \left[\left(-\mathrm{E}[y_i|y_i = s_i] + \mathrm{E}[y_i|y_i \geq s_i]\right)\frac{f_s(s_i)}{1-F_s(s_i)} - 1\right] \\ &\quad \times \left[1 - F_s(s_i)\right]^{n-1} \\ &\quad - (n-1)f_s(s_i)\left(\mathrm{E}[y_i|y_i \geq s_i] - s_i\right)\left[1 - F_s(s_i)\right]^{n-2} \\ &= \left[1 - F_S(s_i)\right]^{(n-2)}\left\{\left[\left(-s_i + E[y_i|y_i \geq s_i]\right)\frac{f_s(s_i)}{1-F_s(s_i)} - 1\right]\left[1 - F_s(s_i)\right]\right\} \\ &\quad - \left[1 - F_S(s_i)\right]^{(n-2)}\left\{(n-1)f_s(s_i)(\mathrm{E}[y_i|y_i \geq s_i] - s_i)\right\} \\ &= \left[1 - F_S(s_i)\right]^{(n-2)}\left\{f_s(s_i)(\mathrm{E}[y_i|y_i \geq s_i] - s_i)(2-n) - \left[1 - F_s(s_i)\right]\right\} \\ &\leq 0\end{aligned}$$

The term outside the curly brackets is clearly positive while $(2 - n) \leq 0$ and $\left[1 - F_s(s_i)\right] \geq 0$ thus the inequality follows. ■

[20] See, Menezes and Monteiro (2003), for the respective proofs.

## References


Adams, W.J., Yellen, J.L., 1976. Commodity bundling and the burden of monopoly. Q. J. Econ. 90 (3), 475–498.

Agência Nacional de Energia Elétrica (ANEEL), 2021. Leilão de transmissão N0 1/2021-ANEEL. Available at https://www.gov.br/aneel/pt-br/assuntos/noticias/leilao-de-transmissao-de-energia-1-2021.

Armstrong, M., 2000. Optimal multi-object auctions. Rev. Econ. Stud. 67 (3), 455–481.

Aryal, G., Charankevich, H., Jeong, S., Kim, D.H., 2023. Procurements with bidder asymmetry in cost and risk-aversion. J. Bus. Econom. Statist. 41 (4), 1143–1156.

Asker, J., Cantillon, E., 2010. Procurement when price and quality matter. Rand J. Econ. 41 (1), 1–34.

Avery, C., Hendershott, T., 2000. Bundling and optimal auctions of multiple products. Rev. Econ. Stud. 67 (3), 483–497.

Bulow, J., Roberts, J., 1989. The simple economics of optimal auctions. J. Political Econ. 97 (5), 1060–1090.

Chakraborty, I., 1999. Bundling decisions for selling multiple objects. Econom. Theory 13 (3), 723–733.

Chakraborty, I., Khalil, F., Lawarree, J., 2021. Competitive procurement with ex post moral hazard. Rand J. Econ. 52 (1), 179–206.

Comisión Nacional de Energía, 2021. Informe técnico definitivo plan de expansión anual de transmisión año 2020. Available at https://www.cne.cl/wp-content/uploads/2021/08/ITD-Plan-de-Expansion-de-la-Transmision-2020-vf.pdf.

Coordinador Eléctrico Nacional, 2021a. Bases de licitación para obras nuevas y obras de expansión del sistema de transmisión nacional 2021–2022. Available at https://www.coordinador.cl/mercados-y-transacciones/licitaciones-de-transmision/.

Coordinador Eléctrico Nacional, 2021b. Informe monitoreo de la competencia en el mercado eléctrico 2021. Available at https://www.coordinador.cl/wp-content/uploads/2022/04/INFORME-Monitoreo-CEN-2021.pdf.

Coordinador Eléctrico Nacional, 2025. Informe monitoreo de la competencia en el mercado eléctrico 2024. Available at https://www.coordinador.cl/wp-content/uploads/2025/04/Informe-Monitoreo-2024.pdf.

De Silva, D., Dunne, T., Kankanamge, A., Kosmopoulou, G., 2008. The impact of public information on bidding in highway procurement auctions. Eur. Econ. Rev. 52 (1), 150–181.

Fudenberg, D., Tirole, J., 1995. Game Theory. MIT Press, Cambridge, Massachusetts, USA.

Goeree, J.K., Offerman, T., 2003. Competitive bidding in auctions with private and common values. Econ. J. 113 (489), 598–613.

Huang, Y.S., Yang, M.S., Ho, J.W., 2022. Bundling decisions for selling multiple items in online auctions. Decis. Anal. 19 (1), 44–62.

Li, S., Sun, H., Yan, J., Yu, J., 2015. Bundling decisions in procurement auctions with sequential tasks. J. Public Econ. 128, 96–106.

Lu, J., Zhao, W., 2023. Intertemporal bundling. Available at SSRN 4387631.

Mares, V., Harstad, R.M., 2003. Private information revelation in common-value auctions. J. Econom. Theory 109 (2), 264–282.

Maskin, E., Riley, J., 2000. Asymmetric auctions. Rev. Econ. Stud. 67 (3), 413–438.

McAfee, R.P., McMillan, J., Whinston, M.D., 1989. Multiproduct monopoly, commodity bundling, and correlation of values. Q. J. Econ. 104 (2), 371–383.

Menezes, F., Monteiro, P., 2003. An Introduction to Auction Theory. Oxford University Press, Oxford, UK.

Menicucci, D., Hurkens, S., Jeon, D.S., 2015. On the optimality of pure bundling for a monopolist. J. Math. Econom. 60, 33–42.

Palfrey, T.R., 1983. Bundling decisions by a multiproduct monopolist with incomplete information. Econ.: J. Econ. Soc. 46, 3–483.

Suzuki, A., 2018. The effect of bundling several contracts on electricity procurement auctions. Waseda Glob. Forum 14, 23–39.

Suzuki, A., 2021. Investigating pure bundling in Japan's electricity procurement auctions. Mathematics 9 (14), 1622.

Wilson, R., 1968. Competitive bidding with disparate information. Manag. Sci. 15 (7), 446–448.