Received: 25 June 2018 | Revised: 4 June 2019 | Accepted: 1 August 2019

DOI: 10.1111/jpet.12396

ORIGINAL ARTICLE

JOURNAL OF PUBLIC ECONOMIC THEORY WILEY

Why is free education so popular? A political economy explanation

Juan A. Correa[1] | Yijia Lu[2] | Francisco Parro[3] | Mauricio Villena[3]

[1]Facultad de Economia y Negocios, Universidad Andres Bello, Santiago, Chile

[2]School of Law, New York University, New York, New York

[3]School of Business, Universidad Adolfo Ibáñez, Santiago, Chile

Correspondence
Francisco Parro, School of Business, Universidad Adolfo Ibáñez, 7941169, Santiago, Chile.
Email: fjparrog@gmail.com

Abstract

This paper analyzes the political support for different funding regimes of education in a one-person, one-vote democracy. We focus the analysis on four systems that have had a preponderant presence in the political debate on education: a private system, a public system that delivers the same resources to each student (universal-free education), a public system that intends to equalize results, and a public system that aims to maximize the output of the economy. We show that a system of universal free education is the Condorcet winner. The level of income inequality and the degree to which income distribution is skewed to the right are key factors behind this conclusion. We also show that the voting outcome of public versus private funding for education depends crucially on the type of public funding under consideration.

1 | INTRODUCTION

Universal free education has become popular in several regions of the world. Western democracies have it at different stages of the educational ladder. European countries, such as France, provide free tuition to European students, and Germany offers free tuition even to international students. Argentina, the Czech Republic, and Greece supply free education at all educational levels. Most of the United States primary and secondary students attend public schools, which provide free education, funded by a mix of federal, regional, and local resources.[1] In other countries, such as Chile, South Africa, and the United Kingdom, where

[1]An extensive cross-country analysis of education's tuition fee schemes can be found in Bentaouet Kattan (2006).

higher education is not free, social movements have pressured the authorities to implement a scheme of universal free education for higher education.[2] In this paper, we give a political economy explanation for the popularity of free education.

A system of universal free education allocates public funds equally across students. This system, however, is not completely consistent with the main implications of a strand of the literature that emphasizes, first, the importance of economic growth to improve living standards and, second, human capital investments as the engine to promote growth (Benhabib & Spiegel, 1994; Hanushek & Kimko, 2000; among others). This branch of the literature points to a system in which public resources for education should be allocated to students with higher skills (so as to maximize aggregate output), relying on alternative instruments for redistribution. Universal free education also implies that public funds are allocated regardless of the student's family income. However, studies such as Samoff (1996) and Larkin and Staton (2001) highlight the importance of equity in the allocation of public resources spent on education. This implies that disadvantaged students should be supported with more resources, which would allow equalizing human capital across students. Hence, universal free education does not point in the direction suggested by these two strands of the literature.

A third strand of the literature suggests that different public funding systems should be implemented at different stages of the educational system. Empirical studies document low returns to interventions targeting disadvantaged adolescents, but high economic returns for remedial investments targeting young disadvantaged children (Cunha & Heckman, 2007; Cunha, James, Lochner, & Masterov, 2006; Heckman, 2008; Heckman & Masterov, 2007). This evidence implies an equity–efficiency trade-off for late child investments but not for early investments (Cunha & Heckman, 2007). Thus, public resources for education should focus on low-income students at earlier stages. However, at later stages, when human capital inequalities are difficult to undo, public resources should be shifted toward high-human capital students so as to maximize output, relying on an alternative instrument for socially desirable redistribution. The popularity of free education at different stages of education is not completely aligned with the implications derived from this third strand of the literature.

Then, why is universal free education so popular in the world? This paper gives a political economy explanation for this popularity. We model a static economy populated by a continuum of heterogeneous agents or *parents*, and each of them has one child and must vote for the funding regime that will finance the education of the child. Parents are heterogeneous in terms of human capital, which equals the family income. The parents' human capital is exogenously given and distributed according to a lognormal distribution function, as in Glomm and Ravikumar (1992) and Becker (1993). We study the Condorcet winner among four funding regimes that frequently appear in the political debate: a private system, a public system that delivers the same resources to all students, a public system that intends to equalize results, and a public system that aims to maximize the output of the economy.

Our analysis shows that a public system that universally invests the same resources in each student is the Condorcet winner in a one-person, one-vote democracy. The intuition behind our

[2]In Chile, the Confederation of Chilean Student Federations (CONFECH), a national body made up of students at Chilean universities, led a series of student protests across the country in 2011. The student movement demanded, among other things, an increase in state support for public universities and free public education. In South Africa, the "Fees Must Fall" movement emerged in 2015 after the government announced an increase in mandatory fees at the universities. Students were placated after the proposal for the increase was dropped. The 2010 United Kingdom student protests were a series of demonstrations held in opposition to the planned increase of the cap on tuition fees by the Conservative-Liberal Democrat coalition government. The biggest demonstration occurred in November 2010, officially known under the phrase of "Fund Our Future: Stop Education Cuts," where thousands of students marched through central London demanding free education.

key finding relies on the lognormal distribution of income, which is skewed to the right. A public system that equalizes outcomes will channel more resources per student to a minority of poor students. The efficiency-oriented system, in contrast, diverts more resources per student to a minority of wealthy students. The majority therefore does not favor public systems that disproportionally benefit a small group of either poor or rich agents, in comparison to the system that equalizes resources across students. In addition, the lognormal income distribution also implies that the median income is below the per capita level. A proportional tax on income that is then redistributed evenly between all students benefits those whose income falls below the mean. Then, the latter agents, who are the majority, prefer the public system that invest the same amount in each student, rather than the private system.

Therefore, our paper provides a political economy explanation for the popularity of universal free education. We show that an ex ante egalitarian public funding system for education is the Condorcet winner when it is confronted by a private system, an ex post egalitarian public system, and an output-maximizing public system. In addition, we show that the voting outcome of public versus private funding for education depends crucially on the type of public funding under consideration. Concretely, we prove that voters might choose a private system when a government proposes as a single alternative either a public system that intends to equalize results or a public system that aims to maximize the output of the economy. Thus, the voting outcome of the public versus private funding systems is not a trivial issue. We also discuss extensions to the baseline model, to show that our main result holds in democracies with a limited degree of either elitism or populism and in a type of top-up education system.

Our work builds upon earlier studies of the political economy of education funding. Creedy and Francois (1990) examine the conditions under which an uneducated majority of individuals support the financing of a proportion of the costs of education through the tax system. Glomm and Ravikumar (1992) analyze the political support for private versus public education, but in their model, voters face only one public funding design. Fernandez and Rogerson (1995) claim that the net effect of public support for higher education is a transfer of resources from poor to rich agents. They show that the underlying factor behind this result is the fact that education is only partially publicly provided. Then, the rich and the middle class may vote for relatively low subsidies to exclude poorer agents from education in the presence of credit constraints to privately finance education. Epple and Romano (1996) study the existence and properties of voting equilibria over public school expenditure in the presence of a private alternative.

More recently, De Fraja (2001) studies the voting equilibrium when voters must choose between two higher education reforms: the imposition of an ability test for admission to a university and a uniform subsidy to university attendance financed by a proportional tax on income. In a similar line, Anderberg and Balestrino (2008) study the voting equilibrium when there are two options to finance higher education in an economy with credit constraints: A subsidy to those who participate in education and a proportional income tax. Borck and Wimbersky (2014) study the political determination of higher education finance. The authors focus their analysis on the factors that might contribute toward higher education reforms from a traditional tax-subsidy scheme to income-contingent loan schemes or graduate taxes.

These previous studies have not analyzed the political support for education funding systems when private education competes with public funding alternatives aiming at equalizing resources, equalizing results, or maximizing output. Including a complete list of public funding, alternatives is important since, as we show explicitly in this paper, the Condorcet winner indeed depends on the specific design for the public funding alternative. In this sense, the analysis developed by Glomm and Ravikumar (1992), who consider a single public funding system, does not contain straightforward implications about the Condorcet winner for the case in which the pool of alternatives for the voters includes several public funding schemes.

The rest of this paper is organized as follows. Section 2 presents the model and derives human capital formation under different education funding systems. Section 3 analyzes the political support for alternative education funding systems. Section 4 discusses extensions to our model. Finally, Section 5 concludes.

2 | THE MODEL

Consider a static economy populated by a continuum of heterogeneous agents or *parents*, each with only one child.[3] Children are differentiated by the human capital they inherit from their parents. This initial human capital of the child is an input for the child's formal education. Parent i's initial human capital, h_p^i, is exogenously given and distributed according to a lognormal distribution function G with parameters μ and σ^2 over support $(0, +\infty)$.[4] We normalize the size of the population to 1.

Children do not make any decisions. They only receive education, which is used to accumulate human capital. Each parent decides how to allocate her income h_p^i between consumption c^i and her child's education y^i. We set labor to 1; thus, an agent's labor earnings equal her human capital. Parents cannot borrow against the future earnings of their children, since there is no capital market in this economy.[5]

All individuals have identical preferences. The preferences are for own consumption and for the total human capital they pass on to their descendants, as in Banerjee and Newman (1991).[6] Specifically, agent i has the following utility function,

$$U(c^i, h_c^i) = \ln c^i + \lambda \ln h_c^i, \tag{1}$$

where c^i is the agent's consumption and h_c^i is the total human capital passed on to the child, discounted by $\lambda \in (0,1)$. The human capital passed on is determined by the following equation:[7]

$$h_c^i = \theta(v^i + y^i)^\gamma (h_p^i)^\delta, \tag{2}$$

which depends upon agent i's human capital h_p^i and the total amount $v^i + y^i$ of resources invested in the education of the child, where v^i are the resources (or voucher) invested in education by the government in the child of agent i and y^i are the resources invested in education by agent i, the parent. The parameter $\theta > 0$ is an exogenous constant. The parameter

[3]Sleebos (2003) reports that the average fertility rate in OECD countries is about 1.6 children per woman. Docquier (2004) shows that there is no clear relation between income and fertility in developed countries. However, a more general model with endogenous fertility rates would be an interesting avenue for future research.

[4]Since Gibrat (1931), the lognormal distribution has been extensively used to describe within- or between-country income distributions. The lognormal distribution has been empirically shown to explain most of the income distribution (see Clementi & Gallegati, 2005; Neal & Rosen, 2000; among others).

[5]Several studies have highlighted capital market imperfections as an important aspect of the investment in human capital (e.g., Aghion & Bolton, 1992; Becker, 1993; Becker & Tomes, 1979; Galor, 2000; Moav, 2002; among others).

[6]A more sophisticated formulation for altruism (Kohlberg, 1976; Loury, 1981; Becker, 1986; Banerjee & Newman, 1991; Becker, 1993, among others) leads to an untractable formulation when comparing different regimes.

[7]The human capital that parents pass on their children can be interpreted either as the initial skills of preprimary students who starts formal education or as the amount of human capital with which a secondary student starts her tertiary education.

$\gamma \in (0,1)$ captures the returns to investment in education and the parameter $\delta > 0$ captures the returns to the parental human capital.

The only difference between the educational systems studied is made by the constraints imposed upon v^i and y^i. Under a purely private system, the government makes no investment in education, so $v^i = 0$. Agent i, therefore, divides her income h_p^i between consumption c^i and private investment in the education of her child y^i, with $h_p^i = c^i + y^i$. Under public education, only the government invests in education, so $y^i = 0$. Since agent i spends nothing on education, all of the post-tax income $(1-\tau)h_p^i$ goes into consumption: $c^i = (1-\tau)h_p^i$, where τ is the tax rate on the agent's income. The total revenue raised by the government is τH_p, where $H_p = \int h_p dG(h)$. This revenue is distributed among the students in the following three ways in the public education systems we study: (a) equally ex ante, with $v^i = v^j, \forall\, i, j$; (b) equally ex post, so that $h_c^i = h_c^j, \forall\, i, j$; and (c) output maximizing, so that $dh_c^i/dv^i = dh_c^j/dv^j, \forall\, i, j$. In all three cases, budget balance requires $\mathbb{E}[v] = \tau H_p$, where $\mathbb{E}$ denotes the expectation operator.

2.1 | The private education system (S1)

In this section, we study the optimal investment in education under a purely private funding system, where the government's investment in education is absent ($v^i = 0, \forall\, i$). Agent i, therefore, chooses c^i and y^i to maximize $U(c^i, h_c^i)$ subject to the technology of human capital formation $h_c^i = \theta(y^i)^\gamma (h_p^i)^\delta$ and the feasibility constraint $h_p^i = c^i + y^i$. The first order condition with respect to y^i is

$$y^i = \left(\frac{\lambda\gamma}{1+\lambda\gamma}\right)h_p^i. \tag{3}$$

Therefore, parents invest a constant fraction $\lambda\gamma/(1+\lambda\gamma)$ of their income in the education of their children. We prove later that the fraction of the income that parents privately invest in education is identical to the majority's preferred tax rate.

2.2 | The public education systems

Now suppose that education is financed publicly. No private acquisition of education is allowed, so $y^i = 0$. Thus, agents consume their after-tax income $c^i = (1-\tau)h_p^i$. Public education is financed by a proportional income tax τ. The resources collected by the government are used to provide education to children. We focus on three different public funding systems. In the first, the government invests an equal amount of money in each student. In the second, the government invests resources to equalize the human capital of the students at the end of the education stage. In the third, the government seeks to maximize the total human capital of the economy.

2.2.1 | The ex ante egalitarian public education system (S2)

In this public system, the government invests the same amount of resources in each student. The subsidy given to each student is denoted by v. Under the constraint that total expenditures must be equal to the total resources collected by the proportional income tax, the equilibrium investment in each student is

$$v = \tau\mathbb{E}[h_p]. \tag{4}$$

Hence, the government gives a flat subsidy to all students. The amount of this subsidy is equal to a fraction τ of the per capita income of the economy. Moreover, since agent i's utility is $\ln(1-\tau)+\gamma\lambda\ln\tau+$ (terms independent of τ), the tax rate τ^i that maximizes agent i's utility is $\tau^i=\gamma\lambda/(1+\gamma\lambda)$. Since τ^i is independent of agent i's characteristics, the same tax rate maximizes all agents' utilities. Therefore, we have that the government chooses $\tau=\gamma\lambda/(1+\gamma\lambda)$, which is the tax rate preferred by all parents.

2.2.2 | The ex post egalitarian public education system (S3)

In this system, the government seeks to remedy initial inequalities in human capital through investments in education that equalize ex post human capital. To do so, the government invests in agents i and j the amounts v^i and v^j, respectively, such that $h_c^i=h_c^j$. Therefore, the relative public investment in students from different families must satisfy $v^i/v^j=(h_p^j/h_p^i)^{\delta/\gamma}$. Taking expectations with respect to j and imposing the balanced-budget constraint $\mathbb{E}[v]=\tau\mathbb{E}[h_p]$, we have that the amount invested by the government on a student from family i is

$$v^i=\tau\mathbb{E}\,[h_p]\left(\frac{(h_p^i)^{-\delta/\gamma}}{\mathbb{E}[h_p^{-\delta/\gamma}]}\right). \tag{5}$$

Therefore, each student receives a proportion of the per capita subsidy delivered under regime S2. This proportion decreases with the initial level of the human capital of the student (or, equivalently, with the family income). Specifically, the proportion of the per capita voucher that each student receives varies according to some measure of the gap between the initial human capital of the student and the average human capital of the economy. Poorer students receive more resources to compensate for their initial lower levels of human capital so that the results of the educational process are equalized across all students.

Additionally, as in the case of an ex ante egalitarian system, the same argument applies to show that the tax rate chosen is $\tau=\gamma\lambda/(1+\gamma\lambda)$.

2.2.3 | The output maximizing public education system (S4)

In the third public system, the government invests the collected resources to maximize the total human capital of the economy. Given this goal, the efficient expenditure is achieved when the marginal product of investment in each student is equalized, that is, $dh_c^i/dv^i=dh_c^j/dv^j,\ \forall\, i,j$. Therefore, the relative amount of resources invested in each family is $v^i/v^j=(h_p^i/h_p^j)^{\delta/(1-\gamma)}$. As we did before, taking expectations with respect to j and imposing the balanced-budget constraint on the government $\mathbb{E}[v]=\tau\mathbb{E}[h_p]$, we obtain[8]

$$v^i=\tau\mathbb{E}[h_p]\left(\frac{(h_p^i)^{\delta/(1-\gamma)}}{\mathbb{E}[h_p^{\delta/(1-\gamma)}]}\right). \tag{6}$$

[8]Equation (6) characterizes a maximum only if the second-order condition holds: $\gamma(\gamma-1)\theta(v^i)^{\gamma-2}(h_p^i)^{\delta}<0,\forall\, i$. This condition holds since we have assumed that $\gamma\in(0{,}1)$.

In this regime, each student receives a voucher that is increasing in the level of the student's initial human capital, since the marginal product of public investment in education is higher in students with a greater initial human capital. Therefore, output maximization requires providing larger subsidies to better-endowed students. As in the previous cases, it is straightforward to show that the tax rate chosen by the majority is $\tau = \gamma\lambda/(1 + \gamma\lambda)$.

3 | POLITICAL SUPPORT FOR THE EDUCATION FUNDING SYSTEMS

In this section, we analyze the political support for different education funding systems in a one-person, one-vote democracy. Concretely, we study the existence and identity of the Condorcet winner among the four funding systems described in Section 2. The game is solved by backward induction. First, the taxes are determined for each system. Then, the systems are compared in pairwise elections and the Condorcet winner is elected.

3.1 | Utility comparison

We first derive the indirect utility $V(h_p^i)$ of an agent i under the four funding systems. In the expressions below, we group the terms to facilitate a comparison of the channels through which the agent's human capital h_p^i impacts the agent's utility. In addition, we discuss each of these channels and assess the ones that matter in our comparison.

$$V^{S1}(h_p^i) = \ln\left(\frac{1}{1 + \lambda\gamma}\right)h_p^i + \lambda \ln \theta + \lambda\delta \ln h_p^i + \lambda\gamma \ln\left(\frac{\lambda\gamma}{1 + \lambda\gamma}\right)h_p^i, \tag{7}$$

$$V^{S2}(h_p^i) = \ln(1 - \tau)h_p^i + \lambda \ln \theta + \lambda\delta \ln h_p^i + \lambda\gamma \ln \tau\mathbb{E}[h_p], \tag{8}$$

$$V^{S3}(h_p^i) = \ln(1 - \tau)h_p^i + \lambda \ln \theta + \lambda\delta \ln h_p^i + \lambda\gamma\left[-\frac{\delta}{\gamma} \ln h_p^i + \ln \tau\mathbb{E}[h_p] - \ln \mathbb{E}[(h_p)^{-\delta/\gamma}]\right], \tag{9}$$

$$V^{S4}(h_p^i) = \ln(1 - \tau)h_p^i + \lambda \ln \theta + \lambda\delta \ln h_p^i + \lambda\gamma\left[\left(\frac{\delta}{1 - \gamma}\right)\ln h_p^i + \ln \tau\mathbb{E}[h_p] - \ln \mathbb{E}[(h_p)^{\delta/(1-\gamma)}]\right]. \tag{10}$$

Human capital influences the utility of an agent through three channels. First, human capital determines the income of the agent and, thus, the agent's consumption. The equilibrium of disposable income for consumption under the private system is $(1/(1 + \lambda\gamma))h_p^i$, and it is $(1 - \tau)h_p^i$ under each of the public systems. We have already shown that the chosen tax rate is $\lambda\gamma/(1 + \lambda\gamma)$. Thus, the amount invested by each parent in the private system equals the taxes paid by them to finance a public system. It follows that the equilibrium consumption level reached by any agent is the same under each of the four education funding systems.

We, therefore, conclude that the impact of a funding system on the disposable income of an agent is not a decisive factor to tilt the balance in favor of one of the funding systems.

Human capital also affects the indirect utility of agents through the production technology of human capital, described by Equation (2). Agents have preferences not only on consumption but also on the human capital they pass on to their children. Thus, the human capital of a parent directly determines the child's human capital and, through this channel, influences the parent's indirect utility. The latter effect is equal to $\lambda\delta\ln h_p^i$ and is identical under the four systems. Hence, neither does this channel play a role in the choice of the education funding system.

The third channel through which human capital affects the utility of an agent is the parental income's impact on the resources for education that the child receives under each of the funding systems. In the private system, parents invest a fixed fraction of their income, as reflected by the term $\ln y^i = \ln(\lambda\gamma/(1+\lambda\gamma))h_p^i$ in Equation (7). Thus, there is a positive relationship between parental income and the resources invested in the student of the corresponding family. The ex ante egalitarian public education system (S2) invests the same resources in each family, as captured by the term $\ln v^i = \ln \tau\mathbb{E}[h_p]$ in Equation (8). Thus, there is no relationship between one family's income and the resources that the system invests in the student from that family. The ex post egalitarian public education system (S3) seeks to equalize ex post human capital. Thus, this system invests more in students from low-income families, generating a negative relationship between parental income and the resources invested by the system in the student. This relationship is expressed by $\ln v^i = -(\delta/\gamma)\ln h_p^i + \ln \tau\mathbb{E}[h_p] - \ln \mathbb{E}[(h_p)^{-(\delta/\gamma)}]$ in Equation (9). The opposite occurs with the efficient system (S4), which invests more in students from high-income families, as expressed by the term $\ln v^i = (\delta/(1-\gamma))\ln h_p^i + \ln \tau\mathbb{E}[h_p] - \ln \mathbb{E}[(h_p)^{(\delta/(1-\gamma))}]$ in Equation (10). Therefore, different systems invest differently in the student of a given family, even though the resources that the family disburses under each of the funding systems are identical.

The previous discussion implies that parents will support the system that invests the most in their children. The private system (S1) and the efficient system (S4) invest more in students from richer families, whereas the opposite occurs under the ex post egalitarian public education system (S3). The ex ante egalitarian public education system (S2) is neutral as it invests exactly the same amount in each student.

As an intermediate step in our analysis, we express Equations (7)–(10) in a simpler and more informative form. To do so, note that the resources invested by each of the systems in a student depend on first and second moments of the income distribution, that is, the average income and how unequally it is distributed over the families. We use the properties of the lognormal distribution to derive an expression for $\mathbb{E}[h_p]$, $\mathbb{E}[(h_p)^{-(\delta/\gamma)}]$, and $\mathbb{E}[(h_p)^{(\delta/(1-\gamma))}]$. For a lognormal distribution, we know that $\mathbb{E}[(h_p)^n] = \exp(n\mu + (1/2)n^2\sigma^2)$ for any $n \in \mathbb{R}$. Therefore,

$$\mathbb{E}[h_p] = \exp\left(\mu + \frac{1}{2}\sigma^2\right), \tag{11}$$

$$\mathbb{E}[(h_p)^{-\delta/\gamma}] = \exp\left(-\frac{\delta}{\gamma}\mu + \frac{1}{2}\left(\frac{\delta}{\gamma}\right)^2\sigma^2\right), \tag{12}$$

$$\mathbb{E}[(h_p)^{\delta/(1-\gamma)}] = \exp\left(\left(\frac{\delta}{1-\gamma}\right)\mu + \frac{1}{2}\left(\frac{\delta}{1-\gamma}\right)^2\sigma^2\right). \tag{13}$$

We substitute (11)–(13) into Equations (7)–(10) and obtain the utility of an agent i as a function of the first and second moments of the income distribution. To do so, we use the fact that $\tau = \lambda\gamma/(1+\lambda\gamma)$ and let $\omega^i = \ln(1-\tau)h_p^i + \lambda\ln\theta + \lambda\delta\ln h_p^i + \lambda\gamma\ln\tau$. Observe that ω^i is the same for all the education funding systems. Thus, we can focus the analysis on the elements of the indirect utility function that are affected by the investment that the funding system makes in the students, as we have already concluded in the earlier discussion.

$$V^{S1}(h_p^i) = \omega^i + \lambda\gamma\ln h_p^i, \tag{14}$$

$$V^{S2}(h_p^i) = \omega^i + \lambda\gamma\left(\mu + \frac{1}{2}\sigma^2\right), \tag{15}$$

$$V^{S3}(h_p^i) = \omega^i + \lambda\gamma\left[-\frac{\delta}{\gamma}\ln h_p^i + \left(1+\frac{\delta}{\gamma}\right)\mu + \frac{1}{2}\left(1-\left(\frac{\delta}{\gamma}\right)^2\right)\sigma^2\right], \tag{16}$$

$$V^{S4}(h_p^i) = \omega^i + \lambda\gamma\left[\left(\frac{\delta}{1-\gamma}\right)\ln h_p^i + \left(1-\frac{\delta}{1-\gamma}\right)\mu + \frac{1}{2}\left(1-\left(\frac{\delta}{1-\gamma}\right)^2\right)\sigma^2\right]. \tag{17}$$

Note that $\sigma = 0$ in a completely egalitarian economy, in which the four systems give agent i the same utility if $h_p^i = \exp(\mu)$; that is, $V^j(\exp(\mu)) = \omega^i + \lambda\gamma\mu$, for all $j \in \{S1,S2,S3,S4\}$. This agent with income $h_p^i = \exp(\mu)$ is the one with the median income of a lognormal distribution. Positive levels of inequality, however, break this indifference between the systems and make the choice of the Condorcet winner nontrivial.

3.2 | Pairwise elections and the Condorcert winner

In this section, we use Equations (14)–(17) to study pairwise voting among the four regimes. Define $h^{Sa,Sb}$ as the income level at which the indirect utilities of the agent under systems Sa and Sb are the same, where $a, b \in \{1,2,3,4\}$. We compute this income threshold for the pairs {S2,S1}, {S2,S3}, and {S2,S4}. For each of these pairwise comparisons involving S2, we assess whether a majority coalition exists to elect S2. We show that in any pairwise election involving S2, this system emerges as the winner.

Using Equations (14)–(17), we obtain

$$h^{S2,S1} = \exp\left(\mu + \frac{1}{2}\sigma^2\right), \tag{18}$$

$$h^{S2,S3} = \exp\left(\mu - \frac{1}{2}\frac{\delta}{\gamma}\sigma^2\right), \tag{19}$$

TABLE 1 Condorcet winner, $\delta > (1-\gamma)$ and $\sigma > 0$

Election	I	II	III	IV	Outcome
{S2, S1}	S2	S2	S1	S1	S2
{S2, S3}	S3	S2	S2	S2	S2
{S2, S4}	S2	S2	S2	S4	S2

$$h^{S2,S4} = \exp\left(\mu + \frac{1}{2}\left(\frac{\delta}{1-\gamma}\right)\sigma^2\right). \tag{20}$$

We examine the cases for which $\sigma > 0$. We divide our analysis into three cases, $\delta > (1-\gamma)$, $\delta < (1-\gamma)$, and $\delta = (1-\gamma)$, since the ranking of the $h^{Sa,Sb}$ terms above change depending on the relative values of δ and γ.[9]

Suppose first $\delta > (1-\gamma)$. It follows that $h^{S2,S3} < h^{S2,S1} < h^{S2,S4}$. Therefore, Equations (18)–(20) divide the population into four groups depending on their income h_p^i: Group I for income level $h_p^i \leq h^{S2,S3}$; Group II for income level $h^{S2,S3} < h_p^i \leq h^{S2,S1}$; Group III for income level $h^{S2,S1} < h_p^i \leq h^{S2,S4}$; and Group IV for income level $h_p^i > h^{S2,S4}$. The median voter is the agent m with an income level $h_p^m = \exp(\mu)$. Thus, this division of the income space implies that the median voter belongs to group II. We analyze the majority voting equilibria in the following pairwise elections: {S2, S1}, {S2, S3}, and {S2, S4}.

Consider first the {S2, S1} election. The indirect utility functions $V^{S1}(h_p^i)$ and $V^{S2}(h_p^i)$ imply that $V^{S2}(h_p^i) \geq V^{S1}(h_p^i)$ for all $h_p^i \leq h^{S2,S1}$. Then, S2 provides a greater level of utility than S1 for all agents in Groups I and II. Thus, these agents with incomes below $h^{S2,S1}$ strictly prefer the ex ante egalitarian public education system (S2) to the private system (S1). Since the median voter is in Group II, it follows that Groups I and II form a majority who prefers S2 to S1. Intuitively, the public system invests a fraction $\tau = \lambda\gamma/(1+\lambda\gamma)$ of the mean income of the economy in each student's education. By contrast, the private system puts a fraction $\lambda\gamma/(1+\lambda\gamma)$ of the family's income into the student's education. Thus, agents with incomes below the mean income prefer S2, since the S2 public system invests more in their children than these agents' investment levels under the private system S1.

Consider next the {S2, S3} election. In this case, we have that $V^{S2}(h_p^i) \geq V^{S3}(h_p^i)$ for all $h_p^i \geq h^{S2,S3}$. Then, agents with an income level above $h^{S2,S3}$ strictly support S2 over S3. Therefore, all agents from Groups II, III, and IV form a majority to elect S2 from the {S2, S3} election. Intuitively, S3 invests more in students from low-income families and less in students from high-income families than S2. Therefore, students from richer families (with $h_p^i \geq h^{S2,S3}$) receive more resources under a public system that delivers a flat subsidy (S2) than under a public system that attempts to equalize ex post results (S3).

Lastly, consider the {S2, S4} election. We have that $V^{S2}(h_p^i) \geq V^{S4}(h_p^i)$, for all $h_p^i \leq h^{S2,S4}$. Then, agents with an income level below $h^{S2,S4}$ strictly prefer S2 over S4. Therefore, agents from Groups I, II, and III form a majority that strictly prefers S2 to S4. Intuitively, in comparison to

[9]The cases $\delta > (1-\gamma)$, $\delta < (1-\gamma)$, and $\delta = (1-\gamma)$ correspond to increasing, decreasing, and constant returns to scale in the production function of human capital. We show that S2 is always the Condorcet winner under each of these cases. However, the political support for system S2 in the {$S2$,$S4$} election becomes more pronounced under increasing returns. The latter is a direct consequence of the fact that, under system S4, resources become much more concentrated on the richest students as returns to scale increase.

TABLE 2 Condorcet winner, $\delta < (1-\gamma)$ and $\sigma > 0$

Election	I	II	III	IV	Outcome
{S2, S1}	S2	S2	S2	S1	S2
{S2, S3}	S3	S2	S2	S2	S2
{S2, S4}	S2	S2	S4	S4	S2

S2, system S4 invests more in students from high-income families at the expense of all of the agents in Groups I, II and III.

Table 1 summarizes the voting outcome in the one-on-one elections {S2, S1}, {S2, S3}, and {S2, S4} for the case $\delta > (1-\gamma)$.

We perform now an analogous analysis for the case $\delta < (1-\gamma)$. In this case, $h^{S2,S3} < h^{S2,S4} < h^{S2,S1}$. This again divides the agents into four groups, depending on income h_p^i: Group I with $h_p^i \leq h^{S2,S3}$; Group II with $h^{S2,S3} < h_p^i \leq h^{S2,S4}$; Group III with $h^{S2,S4} < h_p^i \leq h^{S2,S1}$; and Group IV with $h_p^i > h^{S2,S1}$. By the same analysis as the one above, we can show that a majority coalition exists to support S2 in each of the three pairwise elections (see Table 2).

Lastly, consider the case in which $\delta = (1-\gamma)$. It follows that $h^{S2,S3} < h^{S2,S4} = h^{S2,S1}$, dividing the population into three groups: Group I with $h_p^i \leq h^{S2,S3}$; Group II with $h^{S2,S3} < h_p^i \leq h^{S2,S4} = h^{S2,S1}$; and Group III with $h_p^i > h^{S2,S4} = h^{S2,S1}$. The median income is again in Group II. Then, this is a case in which the private system (S1) and the output maximizing system (S4) generate a subsidy schedule such that the indifference between these systems and the system S2 is observed for the same threshold agent: the one with income $h_p^i = h^{S2,S4} = h^{S2,S1}$. Proceeding analogously to what we did previously, we show in Table 3 the results for this case.

Therefore, we conclude from the results of Tables 1 through 3 that a public funding system that collects taxes to invest the same amount in each student is the Condorcet winner.

3.3 | Public versus private funding for education: The type of public funding matters

We have shown that the ex ante egalitarian public education system S2 is the Condorcet winner in pairwise elections pitching S2 against private system S1 and the other two public systems, S3 and S4. In this section, we explore whether the other two public education funding schemes S3 and S4 also beat private education S1 in pairwise elections. That is, we study the Condorcet winner when the private system is confronted by only one public funding alternative that is different from S2. This analysis will shed light on whether the design and number of public funding alternatives matter for political support of public education over private education.

TABLE 3 Condorcet winner, $\delta = (1-\gamma)$ and $\sigma > 0$

Election	I	II	III	Outcome
{S2, S1}	S2	S2	S1	S2
{S2, S3}	S3	S2	S2	S2
{S2, S4}	S2	S2	S4	S2

Consider first the {S1, S3} pairwise election. In this case, the threshold for the indifference between the systems is

$$h^{S1,S3} = \exp\left(\mu + \frac{1}{2}\left(1 - \frac{\delta}{\gamma}\right)\sigma^2\right). \tag{21}$$

We have three cases: $\delta > \gamma$, $\delta < \gamma$, and $\delta = \gamma$. As we did before, we analyze the nontrivial case in which $\sigma > 0$. Suppose $\delta > \gamma$. Then, we have two income groups: Group I consists of agents with income levels $h_p^i \leq h^{S1,S3}$ and Group II consists of agents with income levels $h_p^i > h^{S1,S3}$. In this case, the voter with the median income belongs to Group II since $\exp(\mu) > h^{S1,S3}$. Equations (14) and (16) imply that $V^{S3}(h_p^i) \geq V^{S1}(h_p^i)$, for all $h_p^i \leq h^{S1,S3}$. Then, all agents with income levels below $h^{S1,S3}$ support the public system S3. Intuitively, the private system (S1) results in greater investment for the wealthier students, whereas the public system (S3) leads to greater investment in the poorer students. Therefore, Group I votes for the public system, whereas Group II votes for the private system. Since the median-income voter belongs to Group II, it follows that the majority chooses the private system in the {S1, S3} election.

Suppose now $\delta < \gamma$. In this case, the median income belongs to Group I because $\exp(\mu) < h^{S1,S3}$. As before, Group I votes for the public system S3, whereas Group II votes for the private system. However, with the median-income voter now in Group I, the majority chooses the public system S3. Lastly, when $\delta = \gamma$, the median income coincides with the threshold $h^{S1,S3}$. Thus, half of the voters support the private system and the other half support the public system, resulting in a tie. Table 4 summarizes these results.

Our analysis shows that when public education is pitched against private education, political support indeed depends on the type of public education under consideration. When the private system (S1) and the ex post egalitarian public education system (S3) is proposed to the voters, the majority votes for the private system when the returns to investment in education are relatively low compared with the returns to endowed human capital, as expressed by the condition $\gamma < \delta$. A greater influence of endowed human capital on the formation of the students' human capital requires that an ex post egalitarian public education system (S3) redistribute even more resources to the poor, since $v^i/v^j = (h_p^j/h_p^i)^{\delta/\gamma}$. Thus, the public resources for education become more concentrated on a minority of low-income students, making the public system S3 less popular than the private system S1 for the majority.

We show next that a similar conclusion results when the private system (S1) and the output-maximizing public system (S4) are the only alternatives for the voters. In this case, the income threshold for the indifference between the systems is

$$h^{S1,S4} = \exp\left(\mu + \frac{1}{2}\left(1 + \left(\frac{\delta}{1-\gamma}\right)\right)\sigma^2\right), \tag{22}$$

TABLE 4 Condorcet winner, {S1, S3} Election

Parameters	I	II	Outcome
$\delta > \gamma$ and $\sigma > 0$	S3	S1	S1
$\delta < \gamma$ and $\sigma > 0$	S3	S1	S3
$\delta = \gamma$ and $\sigma > 0$	S3	S1	S1–S3

TABLE 5 Condorcet winner, {S1, S4} Election

Parameters	I	II	Outcome
$\delta > 1 - \gamma$ and $\sigma > 0$	S1	S4	S1
$\delta < 1 - \gamma$ and $\sigma > 0$	S4	S1	S4
$\delta = 1 - \gamma$ and $\sigma > 0$	S1–S4	S1–S4	S1–S4

for $\delta \neq (1 - \gamma)$. Then, we have Group I with income levels $h_p^i \leq h^{S1,S4}$ and Group II with income levels $h_p^i > h^{S1,S4}$. We again have three cases: $\delta > 1 - \gamma$, $\delta < 1 - \gamma$, and $\delta = 1 - \gamma$. Consider first the case $\delta > 1 - \gamma$. Equations (14) and (17) imply that $V^{S1}(h_p^i) \geq V^{S4}(h_p^i)$, for all $h_p^i \leq h^{S1,S4}$. Thus, agents in Group I support S1 whereas agents in Group II support system S4. The median-income voter is in Group I. Therefore the majority votes for the private system S1. The intuition behind this result again relies on the relative importance of parental human capital on the formation of the human capital of their children. A higher value of δ increases the marginal product of endowed resources for the richer students relative to poorer students. As a result, the S4 public system channels even more resources to a minority of rich students, making the private system S1 more appealing to the majority.

Suppose now $\delta < 1 - \gamma$. In this case, we have that $V^{S1}(h_p^i) \geq V^{S4}(h_p^i)$, for all $h_p^i \geq h^{S1,S4}$. Thus, agents in Group I support S4 whereas agents in Group II support S1. The median-income voter is in Group I and, therefore, the majority now votes for the public funding system S4. In this case, even though the public system S4 invests less in students from poorer families than in richer students, the amount received by the poor students under S4 is greater than the amount received by them under S1. The reason for this is that a relatively lower δ implies that the difference in the marginal product of investment across students is smaller. Hence, differences in the resources delivered across students by the system that aims at equalizing marginal product are not so pronounced as the ones that would be observed under the private system.

Lastly, suppose $\delta = 1 - \gamma$. Then, we have $V^{S1}(h_p^i) = V^{S4}(h_p^i)$, for all $h_p^i \in (0, \infty)$, the public system and the private system lead to the same outcome for each family, resulting in a tie. Table 5 summarizes the results.

The previous analysis again reinforces the important message that the voting outcome of public versus private funding for education depends crucially on the *type* of public funding under consideration. As we have shown, when the public funding alternative employs a design that aims to equalize ex post results or maximize output, the majority may elect private education. We also demonstrate that the introduction of an ex ante egalitarian public funding system can resolve this indeterminacy.

4 | EXTENSIONS OF THE MODEL

In this section, we discuss two extensions to the baseline model. First, we address the case of incomplete democracies, where a fraction of the agents do not participate in politics. Second, we introduce an example to consider the complementarity between private and public education.

4.1 | Incomplete democracies

Our main result hinges upon the assumption that voters fully participate in a democracy in practice. However, voting turnout is never complete. In some democracies, the rich are more likely to participate in politics than the poor; in other democracies, the opposite can be true. We define democracy as incomplete when the voting turnout is less than 100%. An incomplete democracy can be biased toward either the rich or the poor. We define a democracy as "elitist" if it excludes a fraction of the poorest agents of the economy. Analogously, we define a democracy as "populist" if it excludes a fraction of the richest agents of the economy. We show in the appendix that our main result holds for democracies with a limited degree of either elitism or populism. We now discuss the intuition behind this result.

Consider first the case of an elitist democracy, which excludes a fraction of the poorest agents of the economy. Since parents will support the system that invests the most in their children, the poorest parents will support S3 over S2 because the ex post egalitarian system gives more resources to children from low-income families than the system that invests equally across students. Thus, the fact that S2 is preferred to S3 in a complete democracy immediately implies that S2 is also chosen when a number of the poorest agents do not participate in politics. In addition, the poorest parents support S2 over S1 and S2 over S4 in pairwise elections because the ex ante egalitarian system invests more in their children than the private system and the efficient public system. In the appendix, we prove that a limited degree of elitism still leaves S2 as the winner in the $\{S2,S1\}$ and $\{S2,S4\}$ elections.

Consider now the case of a populist democracy, which excludes a fraction of the richest agents of the economy. The richest parents support S1 over S2 and S4 over S2. Both the private system and the output maximizing system invest more heavily in their children than the ex ante egalitarian system. In complete democracies, S2 is elected in pairwise elections $\{S2,S1\}$ and $\{S2,S4\}$. Therefore, S2 would also be supported by the majority when a fraction of the richest parents are excluded from voting. In addition, the richest parents prefer system S2 in the $\{S2,S3\}$ election. The appendix shows that the ex ante egalitarian system still wins the $\{S2,S3\}$ election when a democracy's degree of populism is limited.

4.2 | Private and public education as complements

The analysis so far has assumed that private and public education are perfect substitutes in the human capital formation of students; note that the production technology of human capital is $h_c^i = \theta(v^i + y^i)^\gamma (h_p^i)^\delta$. The perfect substitutability between different systems is a realistic setting to study the political outcome when voters must choose a single alternative from a pool of purely private and public funding schemes.

The case in which public and private education are complements introduces two types of changes in the baseline model developed in Section 2. First, the production technology of human capital must address the complementarity between private and public education. Second, the information flow between private and public players must be precisely stated. Several modeling options arise from these considerations.

We sketch an example that modifies the production technology to show how our analysis can readily accommodate the complementarity between private and public education. Suppose that the educational process has two stages. In the first stage, agents carry out optimal private investment leading to $h_{\hat{c}}^i$, which is the human capital of the student belonging to family i at the end of the first stage. Equations (2) and (3) imply that $h_{\hat{c}}^i = \theta(\lambda\gamma/(1+\lambda\gamma))^\gamma (h_p^i)^{\gamma+\delta}$. In the second stage, politicians present the three public funding alternatives to the voters, who then choose the winner. Suppose the level of the human capital of the student at the end of the first stage becomes her initial human capital for the second stage. Substituting $h_{\hat{c}}^i$ into Equation (2), we obtain

$$h_c^i = \theta^{1+\delta}(v^i)^\gamma \left(\frac{\lambda\gamma}{1+\lambda\gamma}\right)^{\delta\gamma} (h_p^i)^{(\gamma+\delta)\delta}. \tag{23}$$

Note that, compared to Equation (2), this setting could exacerbate or mitigate differences in human capital across families, depending on whether $\gamma + \delta \lesseqgtr 1$. This has implications for the amount of resources that the ex post egalitarian public education system (S3) allocates to students from low-income families and the amount that the efficient system (S4) allocates to students from wealthy families. However, the analysis performed in Section 3.2 still holds once we reparametrize δ as $\tilde{\delta} = (\gamma + \delta)\delta$ and we consider the one-on-one elections that only include the public systems for the second stage.

We have shown one possible way to address the complementarity between private and public education. Interesting avenues for future research include the study of sequential voting, with agents first choosing from a pool of different private education schemes followed by a second-round election to choose from a pool of public funding systems. This could shed light on how the design of public funding schemes can affect agents' choices of private investment in education.

5 | CONCLUSIONS

This paper analyzed the political support for different education funding regimes in a one-person, one-vote political system. We showed that a public system that collects taxes and delivers the same amount of resources to each family is the Condorcet winner. In economies with some degree of income inequality, a system that seeks to equalize or maximize educational outcomes concentrates resources on a minority of the population and, therefore, lacks majority support. In addition, families with an income level below the mean receive more net resources under a public system that employs flat subsidies than under a private system. Therefore, a private system also lacks majority support.

The results of this paper provide a political economy explanation for the observation that governments tend to favor free education for all students (i.e., to spend the same amount on each student). Our paper also highlights the importance of specifying the type of public education under discussion. In particular, we show that voters may favor private education over public education when the latter equalizes or maximizes ex post educational outcomes.

ORCID

Francisco Parroz http://orcid.org/0000-0002-4395-9540

REFERENCES

Aghion, P., & Bolton, P. (1992). Distribution and growth in models of imperfect capital markets. *European Economic Review*, *36*(2–3), 603–611.

Anderberg, D., & Balestrino, A. 2008. *The political economy of post-compulsory education policy with endogenous credit constraints* (CESifo Working Paper Series 2304). Munich; CESifo Group.

Banerjee, A. V., & Newman, A. F. (1991). Risk-bearing and the theory of income distribution. *Review of Economic Studies*, *58*(2), 211–235.

Becker, G. S., & Tomes, N. (1979). An equilibrium theory of the distribution of income and intergenerational mobility. *Journal of Political Economy*, *87*(6), 1153–1189.

Becker, G. S. (1986). Human capital and the rise and fall of families. *Journal of Labor Economics*, *4*(3), 1–39.

Becker, G. S. (1993). *Human capital: A theoretical and empirical analysis with special reference to education* (3rd ed.). Chicago, IL: University of Chicago Press.

Benhabib, J., & Spiegel, M. M. (1994). The role of human capital in economic development evidence from aggregate cross-country data. *Journal of Monetary Economics*, *34*(2), 143–173.

Bentaouet Kattan, R. (2006). *Implementation of free basic education policy* (World Bank Education Working Papers Series No. 7).

Borck, R., & Wimbersky, M. (2014). Political economics of higher education finance. *Oxford Economic Papers*, *66*(1), 115–139.

Clementi, F., & Gallegati, M. (2005). Pareto's law of income distribution: Evidence for Germany, the United Kingdom, and the United States. In A. Chatterjee, S. Yarlagadda, & B. K. Chakrabarti (Eds.), *Econophysics of wealth distributions* (pp. 3–14). Milano: New Economic Windows, Springer.

Creedy, J., & Francois, P. (1990). Financing higher education and majority voting. *Journal of Public Economics*, *43*(2), 181–200.

Cunha, F., Heckman, J., Lochner, L. J., & Masterov, D. V. (2006). Interpreting the evidence on life cycle skill formation. In E. A. Hanushek, & F. Welch (Eds.), *Handbook of the Economics of Education* (pp. 697–812). Amsterdam: North-Holland.

Cunha, F., & Heckman, J. (2007). The technology of skill formation. *American Economic Review*, *97*(2), 31–47.

De Fraja, G. (2001). Education policies: Equity, efficiency and voting equilibrium. *Economic Journal*, *11*(471), 104–119.

Docquier, F. (2004). Income distribution, non-convexities and the fertility: Income relationship. *Economica*, *71*(282), 261–273.

Epple, D., & Romano, R. E. (1996). Ends against the middle: Determining public service provision when there are private alternatives. *Journal of Public Economics*, *62*(3), 297–325.

Fernandez, R., & Rogerson, R. (1995). On the political economy of education subsidies. *Review of Economic Studies*, *62*(2), 249–262.

Galor, O. (2000). Income distribution and the process of development. *European Economic Review*, *44*(4–6), 706–712.

Gibrat, R. 1931. *Les Inégalits Économiques*. Paris: Librairie du Recueil Sirey.

Glomm, G., & Ravikumar, B. (1992). Public versus private investment in human capital: Endogenous growth and income inequality. *Journal of Political Economy*, *100*(4), 818–834.

Hanushek, E. A., & Kimko, D. D. (2000). Schooling, labor-force quality, and the growth of nations. *American Economic Review*, *90*(5), 1184–1208.

Heckman, J. J. (2008). Schools, skills and synapses. *Economic Inquiry*, *46*(3), 289–324.

Heckman, J. J., & Masterov, D. V. (2007). The productivity argument for investing in young children. *Review of Agricultural Economics*, *29*(3), 446–493.

Kohlberg, E. (1976). A model of economic growth with altruism between generations. *Journal of Economic Theory*, *13*(1), 1–13.

Larkin, J., & Staton, P. (2001). Access, inclusion, climate, empowerment (AICE): A framework for gender equity in market-driven education. *Canadian Journal of Education*, *26*(3), 361–376.

Loury, G. C. (1981). Intergenerational transfers and the distribution of earnings. *Econometrica*, *49*(4), 843–867.

Moav, O. (2002). Income distribution and macroeconomics: The persistence of inequality in a convex technology framework. *Economics Letters*, *75*(2), 187–192.

Neal, D., & Rosen, S. (2000). Theories of the distribution of earnings. In A. B. Atkinson, & F. Bourguignon (Eds.), *Handbook of Income Distribution* (Vol. 1, pp. 379–427). Amsterdam: Elsevier North-Holland.

Samoff, J. (1996). Which priorities and strategies for education? *International Journal of Educational Development*, *16*(3), 249–71.

Sleebos, J. (2003). *Low fertility rates in OECD countries: Facts and policy responses* (OECD Labour Market and Social Policy Occasional Papers No. 15).

How to cite this article: Correa JA, Lu Y, Parro F, Villena M. Why is free education so popular? A political economy explanation. *Journal of Public Economic Theory*. 2019;1–19. https://doi.org/10.1111/jpet.12396

APPENDIX

In this appendix, we formally prove that our main result holds for democracies with a limited degree of either elitism or populism. Consider first the percentiles of the income distribution in which the agents with human capital $h^{S2,S1}$, $h^{S2,S3}$, and $h^{S2,S4}$ are located. These agents are indifferent between the funding systems in the corresponding pairwise elections analyzed in Section 3.2. The lognormal income distribution implies that an agent with income h_p^i is located in the $\Phi((lnh_p^i - \mu)/\sigma) \times 100$th percentile of the income distribution, where Φ is the cumulative function of the standard normal distribution. For instance, an agent with income $h_p^i = \exp(\mu)$ is in the $\Phi(0) \times 100\text{th} = 50\text{th}$ percentile of the income distribution. Let $p^{Sa,Sb} \times 100$th be the income percentile of an agent with income $h^{Sa,Sb}$. Then, Equations (18)–(20) imply

$$p^{S2,S1} = \Phi\left(\frac{\sigma}{2}\right), \tag{A1}$$

$$p^{S2,S3} = \Phi\left(\frac{-\delta\sigma}{2\gamma}\right), \tag{A2}$$

$$p^{S2,S4} = \Phi\left(\frac{\delta\sigma}{2(1-\gamma)}\right), \tag{A3}$$

We now use Equations (A1)–(A3) to examine whether the ex ante egalitarian public education system (S2) remains the Condorcet winner in democracies with some degree of elitism or populism.

In Section 3.2, we concluded that all agents with an income below $h^{S2,S1}$ prefer the ex ante egalitarian public education system (S2) over the private system (S1) in a pairwise election. Thus, Equation (A1) implies that $\Phi(\sigma/2) \times 100\% > 50\%$ of voters prefer S2. Suppose an elitist democracy excludes a fraction x of the poorest agents from voting. We can compute the x such that S2 is still the winner of the $\{S2, S1\}$ election[10]:

$$\frac{\Phi(\sigma/2) - x}{1 - x} > 0.5. \tag{A4}$$

Therefore, an elitist democracy that excludes less than $\tilde{x}^1 = 2(\Phi(\sigma/2) - 0.5)$ of the poorest agents still votes for the ex ante egalitarian public education system (S2) in the pairwise election $\{S2, S1\}$.

[10]Remember that the population size was normalized to 1.

We proceed analogously for the other two pairwise elections: {S2, S3} and {S2, S4}. As shown in Section 3.2, all agents with an income above $h_t^{S2,S3}$ prefer the ex ante egalitarian public education system (S2) over the ex post egalitarian public education system (S3) in a one-on-one election. Then, Equation (A2) implies that $100\% - \Phi(-\delta\sigma/2\gamma) \times 100\% > 50\%$ of voters prefer S2 to S3. Then, we can use an equation analogous to (A4) to derive the fraction of the richest agents that could be excluded from voting without affecting the selection of S2 in the {S2, S3} comparison:

$$\frac{1 - \Phi(-\delta\sigma/2\gamma) - z}{1 - z} > 0.5. \tag{A5}$$

Therefore, a populist democracy that excludes less than $\tilde{z} = 2(0.5 - \Phi(-\delta\sigma/2\gamma))$ of the richest agents still elects S2 over S3.

Lastly, we know from Section 3.2 that all agents with an income level below $h_t^{S2,S4}$ prefer the ex ante egalitarian public education system (S2) over the output maximizing system (S4) in a one-on-one election. Therefore, Equation (A3) implies that $\Phi(\delta\sigma/(2(1-\gamma))) \times 100\% > 50\%$ of the voters vote for S2. The equation analogous to (A4) is

$$\frac{\Phi(\delta\sigma/(2(1-\gamma))) - x}{1 - x} > 0.5. \tag{A6}$$

Thus, from Equation (A6) we conclude that an elitist democracy that excludes less than $\tilde{x}^2 = 2(\Phi(\delta\sigma/(2(1-\gamma))) - 0.5)$ of the poorest agents still selects the ex ante egalitarian public education system (S2) in the one-on-one election {S2, S4}.

We show now that the ex ante egalitarian public education system (S2) is still the Condorcet winner in democracies with a limited degree of elitism and populism. Consider first an elitist democracy that excludes less than $min\{\tilde{x}^1, \tilde{x}^2\}$ of the poorest agents of the economy. By construction, the ex ante egalitarian public education system (S2) wins the pairwise elections {S2, S1} and {S2, S4}. Moreover, the ex post egalitarian public education system (S3) invests more resources in students from low-income families. Thus, the fact that S2 is preferred to S3 in a complete democracy immediately implies that S2 is also selected when a number of the poorest agents do not participate in politics. Formally, the political support for system S2 in the {S2, S3} election when x of the poorest agents are excluded from voting is $(1 - \Phi(-\delta\sigma/2\gamma))/(1 - x) \times 100\%$. We have already established that in a complete democracy ($x = 0$), $(1 - \Phi(-\delta\sigma/2\gamma)) \times 100\% > 50\%$. Since $((1 - \Phi(-\delta\sigma/2\gamma))/(1 - x)) \times 100\% > 1 - \Phi(-\delta\sigma/2\gamma) \times 100\% > 50\%$ for any positive value of x, it follows that S2 will also be selected in the {S2, S3} election within an incomplete democracy that excludes less than min $\{\tilde{x}^1, \tilde{x}^2\}$ of the poorest agents. Hence, S2 remains the Condorcet winner even if a fraction of the poorest agents do not participate in elections.

Similarly, consider a populist democracy that excludes less than $\tilde{z}$ of the richest agents. By construction, the ex ante egalitarian public education system (S2) wins the {S2, S3} election. In addition, we know that systems S1 and S4 invest more resources in students from richer families, which makes these funding systems especially popular among the richest agents. We have shown that system S2 wins the one-on-one elections {S2, S1} and {S2, S4} in the context of a complete democracy. Then, it will also win in an incomplete democracy that excludes a fraction of the richest agents. Formally, the political support for system S2 in the {S2, S1} and {S2, S4} elections when a fraction z of the richest agents are excluded from voting is $((\Phi(\sigma/2))/(1 - z)) \times 100\%$

and $((\Phi(\delta\sigma/(2(1-\gamma))))/(1-z)) \times 100\%$, respectively. We have already established that in complete democracies ($z = 0$), $\Phi(\sigma/2) \times 100\% > 50\%$ and $\Phi(\delta\sigma/(2(1-\gamma))) \times 100\% > 50\%$. These two conditions imply that $((\Phi(\sigma/2))/(1-z)) \times 100\% > 50\%$ and $((\Phi(\delta\sigma/(2(1-\gamma))))/(1-z)) \times 100\% > 50\%$, for any positive fraction z. Thus, S2 wins the pairwise elections {S2, S1} and {S2, S4} in a populist democracy that excludes less than $\tilde{z}$ of the richest agents. Hence, S2 remains the Condorcet winner even if a fraction of the richest agents do not participate in elections.